Restoration of Services in Interdependent Infrastructure Systems: A Network Flows Approach

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***Abstract* —** Modern society depends on the operations of civil infrastructure systems, such as transportation, energy, telecommunications and water. These systems have become so interconnected, one relying on another, that disruption of one may lead to disruptions in all. The approach taken in this research is to model these systems by explicitly identifying these interconnections or interdependencies. Definitions of five types of infrastructure interdependencies are presented and incorporated into a network flows mathematical representation, i.e. an interdependent layer network model. Using the lower Manhattan region of New York, USA for illustrative purposes, implementation of the model is shown. First the data requirements are presented with realistic data on the interdependent infrastructure systems of power, telecommunications and subways. Next, a scenario is given that causes major disruption in the services provided by these infrastructures and demonstrates the use of the model in guiding restoration of services. The paper concludes with a discussion of accomplishments and opportunities for future work.

Index Terms — Civil Infrastructure Systems, Emergency Management, Mathematical Programming, Networks.

I. BACKGROUND

Modern society relies on the operations of a set of human-built systems and their processes. The set of systems which is investigated by this research is referred to as civil infrastructure systems. These systems are typically considered to be transportation (including roads, bridges, water and rail), energy (including electric power, gas and liquid fuels), telecommunications (including telegraph, telephone, wireless and internet/digital), and finally, water (including wastewater facilities and water supplies). All civil infrastructures systems rely on a constructed system in order to provide services, such as voice and data transmission or power delivery. In general, each system's components can only be used to support services of their respective group. For example, communications lines cannot be used for energy transmission; water system pipelines are not readily available for energy products such as gas or fuel. However, development of digital communication across power lines for voice and data is in progress [1]. While beneficial from an economic viewpoint, this may increase problems since failure of a single component, such as a power line, would disrupt power and communications.

These systems are so essential that they are commonly referred to as lifelines and are included in the broader set of critical infrastructures defined by the President's Council on Critical Infrastructure Protection (PCCIP). As critical infrastructure systems, they are considered "so vital that their incapacity or destruction would have a debilitating effect on our defense and economic security" [2]. This research focuses on the interconnectedness of these lifeline systems. While all the systems characterized by the PCCIP report are considered critical, some of the systems, such as banking or emergency services, rely upon civil infrastructure systems in order to deliver their services. Therefore, disruption in civil infrastructure systems can cause disruption in these critical infrastructure systems, e.g., disruptions in power and communications after the 2001 World Trade Center (WTC) attack forced the closing of the New York Stock Exchange, part of the banking and finance critical infrastructure [3]. However, this paper will focus only on interdependencies among the civil infrastructure systems.

This paper first presents a discussion of infrastructures and interdependencies. The mathematical representation, i.e. an interdependent layer network (ILN) model, is described, including the formulation of the infrastructure interdependencies. This ILN model maintains the management perspective of these systems as independent entities while including their interconnectedness as a system of systems. This interdependence becomes a issue when disruptions occur. Prior efforts in modeling interdependence have developed artificial structures or hybrid systems, losing the independence of the set of systems, and therefore, reducing the value of the models to system managers. Using the lower Manhattan region of New York, USA, for illustrative purposes, implementation of the model is shown. First the data requirements are presented with realistic data on the interdependent infrastructure systems of power, telecommunications and subways. Next, a scenario is given that causes major disruption in the services provided by these infrastructures. The paper concludes a summary and a discussion of opportunities for future work.

A. Managing Disruptions to Critical Infrastructure Systems

When an event occurs that may cause disruptions to more than one infrastructure system or is considered to be beyond the management capability of normal staff, emergency response organizations are activated. Emergency Response Organizations (EROs) exist not only at the federal, state, county or city level, but within organizations responsible for the operation of the infrastructure systems [4, 5]. For New York City, NY, USA, the ERO is the Office of Emergency Management (NYCOEM); at the state level, it is the Emergency Management Office (NYSEMO); within Consolidated Edison (the principal supplier of power), it is the Corporate Emergency Response Center; for Verizon, a telecommunications provider, it is the Emergency Command Center. No matter the name, each of these emergency response organizations is established for the same basic reasons: to set priorities, coordinate response efforts, collect information and keep informed all relevant parties, both within and external to the organization [6]. As NYCOEM becomes aware of needs, requests are made to responsible agencies or companies. Additionally, coordination of resources is made at NYCOEM as they are made aware of the resources each agency or company has available for response and restoration of services. When a priority is established by federal, state or city government officials, it is the responsibility of NYCOEM to make this priority clear to all member agencies.

The present research focuses on supporting the EROs who exist at the city, county or state level in setting priorities and coordinating activities [7]. Additionally, support is provided for EROs in the organizations responsible for managing civil infrastructure systems in responding to events that disrupt services provided by the systems they manage. The decision makers in both types of ERO are responsible for developing strategies for response and restoration and proposing them for review by stakeholders or regulators both within and external to their organization. Once a strategy has been determined, it is implemented by field personnel. The methodology proposed in this research enables both the independent system perspective for managers of each system, as well as providing the interdependent view for persons charged with setting priorities and directing restoration activities when an event impacts two or more of these systems simultaneously, e.g. the New York City Office of Emergency Management.

II. MODELING INFRASTRUCTURE SYSTEMS

A. The Model in the Context of the Management Of Systems

Each agency or company that owned or managed an infrastructure system developed its own control and monitoring systems. Additionally, evolving technology has led to heterogeneous growth. For example, long distance transmission of electricity is typically done with overhead lines at voltages of 110 to 765 kV. This lack of uniform voltage results in a variety of needs for transformers and switching systems within a single utility. Similarly, in telecommunications systems, there is a range of service available based on the infrastructure (copper wire, fiber optic, etc). As the infrastructure systems grew to cover larger regions and to serve growing populations, more advanced monitoring was required. Greater efficiency was gained in systems such as communications when computers began to aid operators in decision making and control. The use of leased communication lines allowed companies to use an existing infrastructure system instead of using proprietary systems. However, reliance on another companies' systems caused interdependencies.

As noted in *The National Strategy for the Physical Protection of Critical Infrastructures and Key Assets* [8] (referred to later as the *National Strategy*), many emergency managers fail to recognize this "interconnectedness" or interdependence of infrastructures in responding to an incident. Infrastructure management systems did not allow a manager of one system to "see" the operations and conditions of another system. Existing modeling systems also failed to include this "interconnectedness". This research provides a "system of systems" view to better understanding the interdependent nature of these systems with respect to mitigation and post-disruption response and recovery.

B. Modeling of Individual Systems

Many efforts over the years have focused on disruptions affecting single infrastructure systems. These efforts include the Complex Interactive Network Systems Initiative (CIN/SI), a joint endeavor between the Department of Defense, academia and the Electric Power Research Institute (EPRI). Discussion of the CIN/SI program is found in Hasse [9] in the annual reports of the initiative consortia and in Amin [10-14]. Salmeron [15] discussed analytic techniques to mitigate disruptions in electric power grids caused by terrorist attack, but only considered components in the power system and not systems they rely on. Haimes [16] looked at the issue of reducing vulnerability of water systems to willful acts and identified the need for further research in identifying critical points and quantitative methods focused on attack consequences, not likelihood of the disruptive event. Both of these needs are addressed in this research. The National Petroleum Council [17] clearly identified the increased reliance of petroleum and gas systems on information technology and telecommunications. Their report also identified interdependency as one of the most difficult areas to understand. Kuhn [18] provided a quantitative analysis of outages in the phone system and did include power system failure as a cause. Klincewicz [19] looked at the integrated design of computer networks, but made no mention of considerations for the components reliance on power. Chamberland [20] discussed design of multi-technology data networks, but failed to consider interconnectedness to other systems. Cremer [21] looked at issues relating to the physical construction of the Internet, focusing on issues of connectivity and degradation of service, but focused completely on only its system's components. While not exhaustive, these papers and reports show the quantity and breadth of past and on-going work.

As previously noted, each of these systems has essentially evolved independently. However as technology has advanced, each infrastructure system has become interconnected to others. The reliance of any of these systems on power is obvious. Remote monitoring and control systems, essential for safe operation of each, rely on a variety of communication paths. Failures within the communications network in one locale, by whatever cause, may have far reaching effects across many systems.

C. Modeling The System Of Systems

Past research has also studied vulnerability and reliability as they relate to interconnected systems. Haimes and Jiang [22] present a Leontief-based input-output model called the inoperability input-output model (IIM) which enabled the accounting for interconnectedness among infrastructure systems. However, this approach worked at a macroscopic level and while useful for vulnerability assessment, it would be difficult to extend this approach to restoration activities. In a more recent work [23], they continue the development of the IIM and its ability to measure economic impact among various sectors in the economy by analyzing both the initial disruption and the ripple effects. Carullo [24] presents experimental studies in electrical power systems with an embedded communication system for transmission of network conditions. However, this work only looks at control issues due to communication system delay issues. Holmgren [25] also presents issues in power control systems and the associated communication systems. Jha and Wing [26] develop a constrained Markov decision process method to investigate survivability within infrastructures systems which rely on computers and computer networks. While the work refers to critical infrastructures and measuring impacts of disruptions, the work consists of computer network survivability analysis as those networks relate to a specific system, in this case, banking and finance.

Significant effort is being expended in the development of simulations of infrastructure interdependencies. The Department of Homeland Security's Information Analysis and Infrastructure Protection Division is sponsoring the National Infrastructure Simulation and Analysis Center. This program is a partnership between Sandia and Los Alamos National Laboratories and includes the Simulation Object Framework for Infrastructure Analysis, the Urban Infrastructure Suite, and the Interdependence Energy Infrastructure Simulation System projects As simulations they can improve understanding of system response to an event or scenario and can be useful in vulnerability studies. Additionally, the Infrastructure Assurance Center at Argonne National Lab is focused on identifying vulnerabilities to disruptions; assessing the impact of such disruptions on quality of life, economy, and national security; developing effective tools, methods, and technologies that address each phase of the infrastructure assurance cycle, and facilitating coordination efforts between involved parties.

All of these efforts are noteworthy and work to improve understanding. However, as stated earlier, models for the management of interdependent infrastructures need to maintain the view most familiar to their managers, that of independent systems, while also modeling them as a system of systems. During normal operations, the systems can be viewed as operating and being managed independently. However, when a disruption occurs, the interdependencies among these systems must be addressed in order to respond to the disruption and ensure rapid restoration of all services.

III. INFRASTRUCTURE INTERDEPENDENCIES

There are a myriad of publications from federal agencies and other public organizations related to the issue of interdependencies among infrastructure systems. These include the USA Patriot Act [27], *The 2002 National Strategy for Homeland Security* [28] the 1997 report *Critical Foundations - Protecting America's Infrastructure*" [2] by the President's Commission on Critical Infrastructure Protection and the subsequent *Presidential Decision Directive 63, The Clinton Administration Policy on Critical Infrastructure Protection* [29] and *Making the Nation Safer: The Role of Science and Technology in Countering Terrorism* [30]). (General papers on the subject of interdependent infrastructures include [31], [32], [33] and [34]) Also, this discussion now extends globally, for example *Thinking about the Unthinkable: Australian Vulnerabilities to High-Tech Risks* [35] and the November 2004 position paper, *Keeping Canadians Safe* [36].

All of these reports discussed the reliance or dependence of the critical systems but Rinaldi, Peerenboom and Kelly [31] formalized the definitions within this on-going discussion of critical infrastructure interdependencies and defined four classes of interdependency. Due to the number of different types of dependencies and interdependencies, these authors [31] classified the entire family of interrelationships among systems as interdependencies, an approach retained in this paper.

Based in part on [31] and prior work [37], this paper will use the following definitions. A *service* is something made available by the infrastructure for use or consumption. A service may be used by people or by other infrastructures: it is provided in order to meet a real or perceived need. An infrastructure can provide one or more services. *Material* is any physical entity or “substance or substances out of which a thing is or can be made”[38]. Examples include electrons, people, product, and electromagnetic signals. Provision of a service requires activities such as movement, collection, transformation or storage of material. Activities may be initiated at one or many locations and may be terminated at one or many locations. Assuming that traversal of a connection between two intersections requires a set of activities from beginning to end, management activities are necessary when provision of the service requires traversal of more than one intersection.

A *disruption* in an infrastructure is said to occur when one or more of the physical components or one or more of the activities needed to operate a physical component cannot function at prescribed levels. Disruptions may or may not result in service degradation. *Service degradation* is said to occur when the service itself cannot be provided at its prescribed level.

The current research identifies five types of interrelationship between infrastructure systems:

- *Input dependence*: the infrastructure requires as input one or more services from another infrastructure in order to provide some other service.
- *Mutual dependence*: at least one of the activities of each infrastructure in a collection of infrastructures is dependent upon each of the other infrastructures. (An example of mutual dependence involving two infrastructures occurs when an output of infrastructure A is an input to infrastructure B, and an output of infrastructure B is an input to infrastructure A.)
- *Shared dependence*: some physical components or activities of the infrastructures used in providing the services are shared.
- *Exclusive-or dependence*: only one of two or more services can be provided by an infrastructure. Exclusive-or can occur within a single infrastructure system or among two or

more systems.

- *Co-located dependence*: components of two or more systems are situated within a prescribed geographical region.

Collectively, these five conditions—input dependence, mutual dependence, shared dependence, exclusive-or dependence and co-location dependence —will be denoted types of *interdependence*, since all imply that an impact on one infrastructure system is also an impact on one or more other infrastructure systems [37]. These definitions of civil infrastructure systems and their interdependencies form the basis of the mathematical formulations developed in the next section.

IV. THE MODELING PARADIGM: NETWORK FLOWS

Interdependent infrastructures are viewed as networks, with movement of commodities (i.e. material) corresponding to flows and with services corresponding to a desired level of these flows. For ease of representation, each network, or infrastructure system, is defined as a collection of nodes and arcs with commodities flowing from node to node along paths in the network. Activities, physical components and intersections are considered to be contained within a node. Similarly, management activities are not considered in traversal of an arc; they are contained within the arc itself. Fundamentals of network flow problems are fairly uniform within the literature and texts on the subject [39]. For each commodity, each node is either a supply node which is a source for the commodity; a demand node which is a point that requires some amount of the commodity; or a transshipment node which is a point that neither produces nor requires the commodity but serve as a point through which the commodity passes. Arcs may, of course, have limited capacities. Infrastructure systems operate in an environment subject to disruptions. These disruptions could be caused by natural phenomenon, human errors or willful acts. Based upon performance criteria, an infrastructure system can be designed to minimize possible service degradation following a disruption. In addition, once a disruption occurs, alternative ways of restoring service can be determined.

Mathematically, a collection of infrastructure systems is represented as follows. Let I denote the set of infrastructures. Infrastructure $i \in I$ has nodes V^i and directed arcs E^i. Associated with each node $j \in V^i$ is a scalar b^i_j representing its supply or demand. If node $j \in V^i$ is a demand point then $b^i_j < 0$; if it is a supply point then $b^i_j > 0$; and if it is a transshipment node then $b^i_j = 0$. A nonnegative vector of variables, x^i_e, represents the flow on each arc e of the infrastructure. Associated with each arc e in E^i are non-negative scalars of costs c^i_e and capacities u^i_e, where $0 \le x^i_e \le u^i_e$.

Arcs are represented using either the endpoints of the arc or the index of the arc. For a node $l \in V^i$ for some infrastructure $i \in I$, let $\delta^+(l)$ denote the set of arcs in E^i that enter node l and let $\delta^-(l)$ denote the set of arcs in E^i that leave node l. Define $\delta(l) := \delta^+(l) \cup \delta^-(l)$, the set of all arcs incident to node l. Without loss of generality, assume that every supply node has no incoming arcs (i.e., $\delta^+(l) = 0$ if $b^i_l > 0$) and that demand nodes have no outgoing arcs, (i.e., $\delta^-(l) = 0$ if $b^i_l < 0$). Define the set $V^{i,+} \subseteq V^i$ to be the nodes $j \in V^i$ with $b^i_j > 0$ (supply nodes). Sets $V^{i,=} \subseteq V^i$ (transshipment nodes) and $V^{i,-} \subseteq V^i$ (demand nodes) are defined similarly.

A transshipment node j may have a limited capacity, w^i_j, modeled by placing an upper bound on the total flow across the arcs $\delta^+(l)$. Included in the model are *flow conservation constraints* that

(i) for supply nodes ensure that total flow out of the node is no greater than the available supply, (ii) for demand nodes ensure that demand is met, and (iii) for transshipment nodes ensure that flow into the node equals flow out of the node. The structural requirements are modeled by constraints on the capacities of arcs and transshipment nodes.

Network flow models can also be characterized as single or multicommodity. Infrastructures such as water, power, gas and sewer would be single commodity systems, where material moves from one or more supply points, through the set of arcs and nodes, subject to constraints on capacity, and reaches one of more demand points, in an optimal fashion. However, systems like transportation and telecommunications have additional requirements. In these cases, commodities moving across the system have specific origin and destination requirements. For example, passengers arriving at a subway station may each have unique destinations and the needs of each passenger must be met. However, these multiple commodities are not moving independently of each other. These additional requirements of multicommodity systems must be accounted for [39].

As before, let I denote the set of infrastructures. Infrastructure $i \in I$ has nodes V^i and directed arcs E^i. Set O^i represents the set of all origin – destination pairs in infrastructure i. Associated with each origin-destination (O-D) pair, $o \in O^i$ is a market, m_o^i, the amount of a commodity which must flow between that O-D pair. Between each O-D pair is a set of possible paths, P_o. Each path p in P_o is comprised of a subset of the arcs in E. The flow on any path is f and the flows across all the paths in P_o must equal m. The flow x on an arc e is determined by summing the flow on all paths which contain e and is constrained by its capacity, u.

A. Mathematical Representation Of Interdependencies

This section will present the mathematical formulations of the five interdependencies presented in Section III.

1) Input: An infrastructure is input interdependent when it requires as input one or more services from another infrastructure in order to provide some other service. As an example, in the case of a telephone switching station, the switching station itself is a transshipment node within the telecommunications network. However, this same switching station from the perspective of the electrical network is seen as a demand node since it needs an adequate source of electricity to operate. This situation may be represented more formally as follows. Denote the demand node for the switching station in the electrical network to be node j. If there is an adequate flow of electric power into node j, the switching station can function; otherwise, the switching station fails. A connector variable, y, is used in this case to represent the two states of the switching station. If adequate power is available at j, then $y = 1$; if not, then $y = 0$. The phone switching station also has some maximum capacity within the telecommunications network. The station's capacity can be represented as the product of the connector variable y and the rated capacity. When adequate power is available the station can operate to its rated capacity (since $y = 1$). On the other hand, if adequate power is not available then the capacity of the station is 0. The value of the connector is set by the conditions existing in one system, and affects the operating characteristics of a second system. Events affecting the power network that have an effect on node j in turn impact a node in the model of the telecommunications network. The effect on any set of systems can be analyzed in a similar manner. Note that some interdependent infrastructure system failures may result in reducing capacity to some value other than zero. For example, loss of supervisory control systems in a subway system may result in operators exercising greater care and slowing trains. Therefore, the post-disruption capacity may be lower than normal. In this case, the connector variable *y* would shift from 1 to a lower value. The exact effect of each disruption must be evaluated during impact assessment.

In general, input interdependency is represented as follows: Let $D(i,h) \subseteq V^{i,-}$ be the set of nodes in *i* that another infrastructure *h* depends upon (parent nodes). Let $D^i := \cup_{h \in I, h \neq i} D(i,h)$ be the interdependent nodes in infrastructure *i*. The remaining nodes in $V^{i,-}$ will be referred to as the independent nodes. For $j \in D(i,h)$, the binary variable $y_{h,l}^{i,j}$ is the connection between node *j* in infrastructure *i* (where it is a demand node) and node *l* in infrastructure *h*, where it may be either a supply, demand or transshipment node.

Let $C(h,i) \subseteq V^h$ be the set of nodes in h that depend on some other infrastructure *i*, (child nodes) and let $C^h := \cup_{i \in I, i \neq h} C(h,i)$ .Without loss of generality, all nodes have been disaggregated to the point where, given infrastructures *i* and *h*, and node *j* in $D(i,h)$, there is a unique node *l* in $C(h,i)$ such that $y_{h,l}^{i,j}$ is defined, and given infrastructures *i* and *h* and node *l* in $C(h,i)$, there is a unique node *j* in $D(i,h)$, such that $y_{h,l}^{i,j}$ is defined. Let $F(i,h)$ be the set of ordered pairs (*j,l*) associated with node *j* in $D(i,h)$ and node *l* in $C(h,i)$ for each $y_{h,l}^{i,j}$.

Input dependency changes the basic network flow model as follows. Following a disruption, the flow into demand nodes may be insufficient. The slack variable, *s*, represents the shortfall. The constraint for all demand nodes becomes

$$s_j^i + \sum_{e \in \delta^+(j)} x_e^i = -b_j^i \qquad \forall j \in V^{i,-}, \forall i \in I \tag{1}$$

and for multicommodity systems is

$$\sum_{p} f^i_{o,p} + s^i_o = m^i_o \qquad \forall o \in O^i,\ \forall p \in P\ \forall i \in I \tag{2}$$

and in the objective function, the weighted slack is:

$$\sum_{i \in I} \sum_{j \in V^{i,-} \setminus D^i} k^i_j s^i_j \text{ for demand nodes} \tag{3}$$

and

$$\sum_{i \in I} \sum_{j \in V^{i,+}} k^i_j s^i_j \text{ for supply nodes.} \tag{4}$$

The existence of slack at an interdependent node (parent system) acts as a control switch for the connector variable, *y*:

$$s^i_j \le (1 - y^{i,j}_{h,l})(-b^i_j) \qquad \forall (j,l) \in F(i,h),\ \forall i,h \in I, i \ne h \tag{5}$$

When *s* is greater than 0, *y* must be 0 in order to satisfy this constraint.

The following component of the objective function with its weighting factor, *k*, works to shift available commodities to or away from parent demand nodes in interdependent systems and tends to push y back to 1:

$$\sum_{i \in I} \sum_{j \in D^i} \sum_{\substack{h \in I \\ h \ne i}} \sum_{\substack{(j,l) \in \\ F(i,h)}} k^i_j (-b^i_j)(1 - y^{i,j}_{h,l}) \tag{6}$$

When a parent node has unmet demand, the corresponding capacity of its child node in the dependent system is reduced. The following constraints are required:

For supply nodes -

$$\sum_{e \in \delta^-(l)} x^h_e \le b^h_l y^{i,j}_{h,l} \qquad \forall (j,l) \in F(i,h) \text{ with } b^h_l > 0, \forall i,h \in I, i \ne h \tag{7}$$

For transshipment nodes -

$$\sum_{e\in\delta^+(l)} x_e^h \le w_l^h y_{h,l}^{i,j} \qquad \forall (j,l)\in F(i,h) \text{ with } b_l^h = 0, \forall i,h\in I, i\neq h \tag{8}$$

For demand nodes –

$$s_l^h + \sum_{e\in\delta^+(l)} x_e^h \le -b_l^h y_{h,l}^{i,j} \qquad \forall (j,l)\in F(i,h) \text{ with } b_l^h = 0, \forall i,h\in I, i\neq h \tag{9}$$

While modeling input interdependency as a node-to-node connection is generally the case, there may be instances where a node-to-arc connection is more appropriate. In our implementation, this node-to-arc input interdependency occurred with the tracks of the subway system. These tracks receive power from DC rectifiers and transfer the power to the subway train motors. In this case, the rectifier will be treated as the parent node in the power system and the subway tracks are the child dependency arcs with the flow being measured along the rails (arcs) is passengers.

Similar to the discussion for the child node set $C(h,i)$, let $G(h,i)\in E^h$ be the set of arcs in *h* that depend on some other infrastructure. As before, given infrastructures *i* and *h* and node *j* in *D(i,h),* there is at most one arc *e* in *G(h,i)* such that $y_{h,e}^{i,j}$ is defined. Let *M(i,h)* be the set of ordered pairs associated with node *j* in *D(i,h)* and arc *e* in *G(h,i)* for each $y_{h,e}^{i,j}$. A new constraint, similar to equation (5), takes the form

$$s_j^i \le (1-y_{h,e}^{i,j})(-b_j^i) \qquad \forall (j,e)\in M(i,h),\ \forall i,h\in I, i\neq h\,. \tag{10}$$

In the objective function, the following term, similar to equation (6) is added

$$\sum_{i\in I}\sum_{j\in D^i}\sum_{\substack{h\in I\\ h\neq i}}\sum_{\substack{(j,e)\in\\ M(i,h)}} k_j^i(-b_j^i)(1-y_{h,e}^{i,j})\,. \tag{11}$$

and for the child system arcs, the following is added to control capacity as the parent demand changes

$$x_e^h \leq u_e^h y_{h,e}^{i,j} \qquad \forall (j,e) \in M(i,h), \forall i,h \in I, i \neq h \tag{12}$$

2) Mutual Interdependence: A collection of infrastructures is said to be mutually interdependent if at least one of the activities of one infrastructure system is dependent upon any other infrastructure system and at least one of the activities of this other infrastructure system is dependent upon the first infrastructure system. Mutual dependence is simply the case of two input dependencies between infrastructures A and B. At one location, infrastructure A is dependent on an output of infrastructure B, and at some other location, infrastructure B is dependent on an output of infrastructure A. Mathematically, this would result in two sets of equations 1 though 9 (or equations 10-12) as appropriate to the situation, one set for each pair-wise connection.

3) Shared: Shared interdependence occurs when some physical components and/or activities of the infrastructure used in providing the services are shared. Phone lines could be considered in the shared interdependency. For example, phone lines carry two types of calls, incoming and outgoing. Each cable section has a maximum capacity. For example, if the capacity of some section is 50, this could be 50 incoming calls or 50 outgoing calls or some combination totaling 50.

Mathematically, let n_n represent each commodity in the set of all commodities, N, and $x_{e,nn}^i$ be the amount of commodity n_n carried across arc e of infrastructure i. Define the set $R^i \subseteq E^i$ to be those arcs $e \in E^i$ that carry multiple commodities. Additionally, arc e has total capacity u. Therefore, the shared interdependency would add a constraint of the form:

$$\sum_{n_n \in N} x^i_{e,nn} \leq u^i_e \tag{13}$$

Similarly for nodes, their capacity is limited to the sum of flow of all commodities into the node. Constraints of this form will have to exist for all arcs and nodes that carry multiple commodities.

4) Exclusive-or: When multiple services share infrastructure component(s), but the component can only be used by one service at a time, exclusive-or interdependence occurs. In the first few days following the WTC attacks, streets (i.e., shared components) could not be used by both the emergency response personnel and financial district workers. This conflict had to be resolved prior to reopening the New York Stock Exchange [40]. Exclusive-or interdependencies are modeled by selecting additional constraints to restrict flow to one commodity or the other.

A binary variable $r^i_{e,nn}$ will indicate whether or not commodity n_n is being carried on arc e in infrastructure i (r=1 means the arc is carrying commodity n_n, 0 its not). Define the set $T^i \subseteq E^i$ to be those arcs $e \in E^i$ which are subject to the exclusive-or interdependency. Mathematically, this is

$$x^i_{e,nn} \leq u^i_e r^i_{e,nn} \tag{14}$$

$$\sum_{nn \in N} r^i_{e,nn} \leq 1 \tag{15}$$

5) Co-located: The co-located interdependency occurs when any of the physical components or activities of the civil infrastructure systems are situated within a prescribed geographical region. Managers of individual infrastructure systems would identify the components of their respective system at or near the site of the incident which may have been affected by the event and an interface such as a geographic information system (GIS) would aid the managers in identifying these potentially affected components. Based on inspection or follow-up reports, the condition of these components will be adjusted by changing their available supply, demand or capacity as appropriate. This adjustment of component characteristics is the only mathematical representation of the co-located interdependence, since the mere proximity of components does not mean they will fail and this model does not consider likelihood of failure. Each case involving the co-located interdependency must be considered separately, since the type of event will affect the impact on nearby systems. For example, a water main break could have adverse effects on power and phone lines, but little impact to a gas line. On the other hand, a steam line break may have little effect on a power or phone line but might result in softening of non-metallic gas piping.

B. The Objective Function and Constraints of an Interdependent Layered Network Model

This research has developed a formal, mathematical representation of the set of civil infrastructure systems that explicitly incorporates the interdependencies among them and will be referred to as an interdependent layered network (ILN) model. This ILN is a mixed-integer, network-flow based model which has been implemented in software that enables the resulting model to be exercised. This interdependent layered network model in standard form is given by:

Minimize

$$\sum_{i\in I}\sum_{e\in E^i} c_e^i x_e^i + \sum_{j\in V^{i,-},V^{i,+}\setminus D^i} k_j^i s_j^i + \sum_{i\in I}\sum_{o\in O^i} k_o^i s_o^i + \\ + \sum_{i\in I}\sum_{j\in D^i}\sum_{\substack{h\in I\\ h\neq i}}\sum_{\substack{(j,l)\in\\ F(i,h)}} k_j^i(-b_j^i)(1-y_{h,l}^{i,j}) + \sum_{i\in I}\sum_{j\in D^i}\sum_{\substack{h\in I\\ h\neq i}}\sum_{\substack{(j,e)\in\\ M(i,h)}} k_j^i(-b_j^i)(1-y_{h,e}^{i,j}) \tag{16}$$

Subject to

$$\sum_{e\in\delta^-(j)} x_e^i \le b_j^i \qquad \forall j\in V^{i,+} \text{ and } \forall i\in I \tag{17}$$

$$s_j^i + \sum_{e\in\delta^+(j)} x_e^i = -b_j^i \qquad \forall\, j\in V^{i,-} \text{ and } \forall i\in I \tag{18}$$

$$\sum_{e\in\delta^+(j)} x_e^i - \sum_{e\in\delta^-(j)} x_e^i = 0 \qquad \forall j\in V^{i,=} \text{ and } \forall i\in I \tag{19}$$

$$\sum_{e\in\delta^+(j)} x_e^i \le w_j^i \qquad \forall j\in V^{i,=} \text{ and } \forall i\in I \tag{20}$$

$$\sum_{e\in\delta^-(j)} x_e^h \le b_j^h y_{h,j}^{i,l} \qquad \forall (l,j)\in F(i,h) \text{ with } b_j^h>0, \forall i,h\in I, i\neq h \tag{21}$$

$$s_j^h + \sum_{e\in\delta^+(j)} x_e^h = -b_j^h y_{h,j}^{i,l} \qquad \forall (l,j)\in F(i,h) \text{ with } b_j^h<0, \forall i,h\in I, i\neq h \tag{22}$$

$$\sum_{e\in\delta^+(j)} x_e^h \le w_j^h y_{h,j}^{i,l} \qquad \forall (l,j)\in F(i,h) \text{ with } b_j^h=0, \forall i,h\in I, i\neq h \tag{23}$$

$$s_l^i \le (1-y_{h,j}^{i,l})(-b_l^i) \qquad \forall (l,j)\in F(i,h),\ \forall i,h\in I, i\neq h \tag{24}$$

$$s_j^i \le (1-y_{h,e}^{i,j})(-b_j^i) \qquad \forall (j,e)\in M(i,h),\ \forall i,h\in I, i\neq h \tag{25}$$

$$\sum_{p\in P_o} f_{o,p}^i + s_o^i = m_o^i \qquad \forall o\in O^i, \forall i\in I \tag{26}$$

$$\sum_{\substack{o\in O^i, p\in P_o\\ p \text{ containing } e}} f_{o,p}^i \le u_e^i y_{h,e}^{i,l} \qquad \forall (l,e)\in M(i,h)\ \forall i,h\in I, i\neq h \tag{27}$$

$$\sum_{n_n \in N} x_{e,nn}^i \le u_e^i \qquad \forall e \in R^i, i \in I \tag{28}$$

$$x_{e,nn}^i \le u_e^i r_{e,nn}^i \qquad \forall e \in T^i, i \in I, n_n \in N \tag{29}$$

$$\sum_{nn \in N} r_{e,nn}^i \le 1 \qquad \forall e \in T^i, i \in I \tag{30}$$

$$\sum_{\substack{o \in O^i, p \in P_o \\ p \text{ containing } e}} f_{o,p}^i \le u_e^i \qquad \forall e \in E, i \in I \tag{31}$$

$$\sum_{\substack{o \in O^i, p \in P_o \\ p \text{ containing } e \\ e \in \delta^+(j)}} f_{o,p}^i \le w_j^i \qquad \forall j \in V^=, i \in I \tag{32}$$

$$x_e^i \le u_e^i \qquad \forall i \in I, \forall e \in E^i \tag{33}$$

$$x_e^i \ge 0 \qquad \forall e \in E \text{ and } i \in I \tag{34}$$

$$f_{o,p}^i \ge 0 \qquad \forall o \in O^i, \forall p \in P_o, i \in I \tag{35}$$

$$s_j^i \ge 0 \qquad \forall i \in I \text{ and } \forall j \in V^i \tag{36}$$

$$y_{h,j}^{i,l} \qquad \text{binary, } \forall (l,j) \in F(i,h),\ \forall i,h \in I, i \ne h \tag{37}$$

$$y_{h,e}^{i,l} \qquad \text{binary, } \forall (l,e) \in M(i,h),\ \forall i,h \in I, i \ne h \tag{38}$$

The objective function of this interdependent layered network model (equation (16)) combines the minimum cost terms from traditional network flow models, both single and multicommodity, and the weighted slacks described in equations (3) for independent demand nodes and (4) for supply nodes on page 15, Section IV.A. Additionally, equations (6) and (11), from pages 16 and 17 respectively, are included with their own weighting to shift available commodities to or away from parent demand nodes in interdependent systems. In the constraint set, equations 18 and 26 are added as noted in Section IV.A to represent the shortfall that may be caused by the disruption. The constraint set also includes the flow conservation and capacity constraints from single and multicommodity network flow models, equations 17, 19, 20, 28, 29, 30, 31, 32 and 32. Additionally, equations (24) and (25), discussed earlier as equation (5) and (10), are included as the control switch of the connector variable, y. When there is unmet demand at a parent node, this effect is reflected in the operation of the child node among interdependent systems, as described with equations 21, 22, 23 and 27 (Section IV.A equations (7), (8) (9) and (12)). Shared interdependency constraints and exclusive-or constraints are added as appropriate to the particular systems being modeled and the disruption considered.

V. IMPLEMENTATION

The foregoing presents a formulation of an interdependent layered model in the context of interdependent critical infrastructure systems. The section to follow will discuss its implementation in terms of data requirements, software to run the model, and a human-computer interface for interacting with the model.

A. Data Requirements

In constructing the data set for these systems, the modeler must be provided with both the physical layout of the system and the supplies, demands and capacities for the components of the system. However, systems like transportation and telecommunications have additional requirements. As stated earlier, commodities moving across the system have specific origin and destination requirements. For example, passengers arriving at a subway station may each have unique destinations and the needs of each passenger must be met. However, these multiple commodities are not moving independent of each other. In this case, the modeler not only needs the physical layout and capacities of system components, but the origin and destination requirements (commonly referred to as an O-D matrix) for the system.

1) Power System: In general, power is produced at generating stations at voltages in the range of 13.8 – 22kV. It is transformed up to 345 – 500kV for transmissions, although some higher or lower voltage transmission systems exist. It is reduced in steps for distribution and is delivered to most homes and businesses as 120V and 220V.

Typically, the U.S. power distribution is of a radial design from the substation. Each feeder distributes power to a portion of the service area. Transformers along the feeder provide 120/220V service to homes and businesses. Transformer failures will cause loss to a few homes or businesses, feeder disruptions to a greater number, but only in the sub-area serviced by the feeder. In developing the 120/220V service grid for Consolidated Edison, a different design was used.

In Manhattan, the high voltages of transmission are first transformed down to an intermediate voltage of 69 – 123 kV and further reduced at area substations to 13.5 kV. The power is distributed for the substation on a network of 8 to 28 feeders. Each substation serves two service areas, each area having its own set of feeders. So there might be as many as 56 feeders exiting a substation.

The power system below 60th Street in Manhattan contains 15 substations covering 30 service areas. Each substation has an average capacity of 400 MVA. Each feeder has a capacity load of 15 MVA. Along each feeder 20 to 40 transformers reduce feeder voltage to 120-220V. The output of several hundred transformers are connected in an elaborate 120/220 sub-network.

The system is designed to maintain 120/220V service throughout the area even if as many as 2 feeders fail. This design provides for extremely high reliability. At some key facilities, such as hospitals, phone switching centers, etc. these feeders will each provide power to sets of transformers at that location again assuring highly reliable service. Non-key facilities are connected directly to the 120/220V grid.

The tracks of New York's subway system are powered from DC rectifiers, transforming feeder voltage of 13.5kV ac to 600V dc Each rectifier has a capacity of about 3MW and at least two rectifiers power at each track section (typically 100-1500 ft.). Each rectifier has sufficient capacity to power its respective track section, so the additional rectifiers(s) are providing redundancy and reliability. Each rectifier is connected to 2 feeders and each feeder is capable of carrying the full demand of the rectifier.

The geographic boundaries of the service areas differ from the actual as the model is intended to provide a realistic view because actual data is not available due to security concerns. Each substation distributes power along 8-24 feeders to 17 phone switching centers, 178 ac/dc rectifiers for the subways and service to all residences and businesses in the area. The 120/220V grid was modeled in an aggregate fashion since none of the other major facilities (phone switching centers or subway tracks) relied on it to provide service. Instead of 20-40 transformers along the feeders, a transformer (node) was placed at the ends of the feeder. The 110/220V demands of a service area were aggregated into a single node fed from all the feeder ends. Blocks in the study were then classified residential or commercial. Census data for the 1892 blocks were plotted in bins of 120. It was noted that 633 of 1892 blocks (33%) had populations less than 120. These blocks were classified commercial; the remaining blocks residential. Power demand for a residential block is 2.33 MW and commercial is 10 MW.

2) Model of Telecommunications: The phone system is of a spoke and hub construction. Customers may be connected via a Controlled Environment Vault (CEV) by a distribution cable serving dozens of customers. Distribution cables are combined at the CEV into a feeder cable. Each feeder typically carries thousands of lines. The feeder enters a central office (CO) at the cable vault. In many areas of Manhattan, customers are connected directly to the central office. In the case of 140 West Street, there were 300,000 wire pairs in the cable vault. From a CO, calls are digitally switched to the intended destination, either directly or via a CEV if the call originates and terminates with the service area of the CO. In high density urban areas, like Manhattan, one CO may be connected to other nearby CO's via a trunk line. If the call is for a destination outside the CO and nearby CO's, it is directed to the tandem. Tandems connect to other tandems and to the national and international networks. Seventeen CO's and service areas were included in the model for Lower Manhattan.

Calls are modeled as multicommodity flow problems [39] within a certain market (number of calls) between each origin and destination (O-D) pair. Within the model, since there were no instances of systems that would fail when phones fail, the model only looked at O-D modeling between CO's and any outages would be reflected as the entire service area of the CO. For each O-D pair, a set of paths must be generated and the set of arcs comprising each path is additional data for the model. In the case of the phone system, two paths exist between most central offices.

If cellular service was to be incorporated in the analysis, a tower would be used in the place of a CEV. Cellular calls are connected to the tower and are then transferred via landline to the cellular company CO.

3) Model of the Subway System: The New York City subway system serves 7,000,000 users per day. Information regarding station and track locations was taken from existing system maps from the Metropolitan Transportation Authority. Stations are individual nodes and local and express tracks are arcs.

Similar to phone, passengers are modeled as a multicommodity flow problem. Modeling the flow through the system starts with determining an O-D matrix for a time period of concern. Data was received from the MTA including turnstile counts from each station in the system and data from an annual cordon count. The cordon count is an hour by hour measure of the passenger flow in and out of Manhattan from above (north of 60^{th} St.) from Queens and from Brooklyn. The morning commute is considered by the MTA to be the period from 6am- 9 am and the evening from 4pm – 9pm.

Travel times were determined between each station pair based on track length, an average train speed of 45 mph and 20 seconds for the time stopped at a station. For trips which required transfers at a station from one line to another, the arcs were included in the path with a travel time of one half the advance for the train line the person was transferring to. (The advance is the time between train arrivals; if a line has 20 trains per hour, the advance is 3 minutes.)

Using methods outlined in Fricker [41], an O-D matrix was developed for the 115 stations included in the model. Within the dataset provided by the MTA, no trips originated or terminated from Brooklyn on the B or D trains. This was due to bridge maintenance during the time frame. Paths for each O-D pair were also developed. The path set included the shortest path and alternatives that did not exceed 150% of the shortest path time to a maximum of three paths. For example, if the shortest path for an O-D pair was 20 minutes, up to two alternative paths would be generated as long as the travel time on the alternatives did not exceed 30 minutes. The set of arcs comprising each path are included within the data for the model. Line capacities were based on MTA data.

B. Computing Requirements

1) Size of Problem and Runtime: The attributes of the power, phone and subway systems with the level of detail described in Section IV.A comprise a database of 6.56Mb. The ILN from Section IV.B for the Manhattan example contains 36,485 variables and 25,887 constraints. Using a laptop computer with a Pentium M processor operating at 1.6 GHz and 1 GB of RAM, the undisrupted ILN has a solution time of 16.4 seconds, determining the flow of power, phone calls and subway passengers along the various arcs, within the capacity constraints. Following a disruption like the one to be presented in section V.C, the ILN determines a solution in 4.78 seconds, before restoration efforts begin. Each of the four cases presented in section V.C was solved in less than one second, since the model was only using a subset of the original data. The NP-hard problem of node packing can be reduced to ILN, so the complete ILN model is also NP-hard. Without interdependencies, the problem decomposes into several polynomially solvable single and multicommodity network flow problems. Our results show that the complexity coming from the interdependencies can be handled effectively for realistic ILN problems.

2) Software Requirements: The model has been implemented in AMPL [42] using the CPLEX [43] solver. All of the model’s data is stored in a Microsoft Access database. The database contains the component attributes such as a name, their capacity and their priority, as well as spatial attributes, such as location and length. These spatial characteristics are generated automatically by the GIS software, ESRI's ArcGIS [44] in this case.

3) The User Interface: A geographic information system (GIS) was selected as the user interface as this seemed to be the most natural method of displaying systems and determining affected areas to ERO personnel. The interface allows the operator to update the conditions of the components of the set of systems modeled, to add temporary systems during restoration, and to display areas affected by an inability to meet demands.

C. Scenario

In order to demonstrate the model's usefulness and since our work was related to the September 11 attacks on the World Trade Center (WTC), a scenario with damage of a magnitude similar to the effects of those attacks was developed. After the collapse of the North and South towers on the morning of September 11 and the collapse that afternoon of WTC Building 7 on Vessey St, Consolidated Edison (Con Ed), Verizon and the Metropolitan Transit Authority were faced with the following conditions. For Con Ed, the collapse of WTC 7 resulted in the loss of a power substation and a resulting power outage south of Canal St., an area which included the Verizon switching center on West St, the American Stock Exchange and the New York Stock Exchange. Con Ed restored power throughout most of the area in 8 days using 80 temporary generators and miles of enclosed shunts (shunts are power lines run along streets which connected working portions of the power grid in southern Manhattan to the feeders that had been distributing the power from the destroyed substation in order to restore power).

For Verizon, the power outage would not have normally affected operations due to the presence of emergency generators for the building. However, this was not normal circumstances. Broken water mains and firefighting operations flooded the basement of the building preventing operations of emergency generators there while the extensive dust and debris outside the building made the remaining generator inoperable. Falling debris from all three buildings severed cables leading to and from the switching center and the cable vault where all the feeder cables entered the building was flooded. These combined to result in loss of 300,000 voice lines and 4.4 million data lines. Verizon connected temporary phone lines to the digital switches in the upper floors on the 140 West St office, ran the lines down along the outside of the building and then connected them to undamaged lines approximately one block from the building along Manhattan streets (any temporary line for power or phone will be referred to as a shunt).

The MTA temporarily lost power to all the subway stations south of Canal St due to the power outages with the long term impact being the destruction of the Cortlandt St station on the 1,9 lines located under the World Trade Center. Service south of the WTC complex on the 1,9 line was not available until the station was reconstructed.

To exercise the model, the following scenario has been developed. Some event (no matter the cause) has resulted in the collapse of the Brooklyn – Battery Tunnel. Power and phone lines as well as subway tracks passing over the area are either damaged or severed. The electrical substation at the southern end of Manhattan is also damaged with repairs estimated to take 4 to 6 weeks. The resulting outage affects the area west of Broadway, north to nearly Vessey St and the area east of Broadway, north to Fulton St. Also affected by the power outage is the phone central office between Bowling Green and Battery Place. Other directly affected points of concern are the New York Stock Exchange (NYSE) and the Goldman Sachs offices at Broad St and William

St. as well as the residential and commercial customers. An overview map of Manhattan showing the affected area and a detailed map are shown in figures 1 and 2.

(insert figure 1 here)

(Insert figure 2 here)

Within the subway system, this power outage impacts the Rector St station on the R and W lines; the Wall St, Bowling Green and Fulton St. stations on the 4 and 5 lines; the Wall St and Fulton St. stations on the 2 and 3 lines, the Rector St. and South Ferry St stations on the 1 and 9 lines; and the Broad St and Fulton St stations on the J, M and Z lines, as well as all connecting local and express tracks (shown in Figure 3)

(insert Figure 3 here)

If they did not have emergency backup power supplies, the power outage to the two phone central offices would have resulted in a loss of phone service over a similar area to the power outage. Simulating the WTC event, backup power at the CO at Bowling Green / Battery Place is lost, resulting in a loss of phone service to the Merrill Lynch and Lehman Brothers offices along Vessey St. Additionally, the trunk line between the Bowling Green central office and the central office along Fulton St is severed by the event. This is significant because this trunk line is in the primary or alternate communication path to the NYSE for the trading offices of Merrill Lynch and Lehman Brothers in southern Manhattan and Morgan Stanley along Broadway between 48th

and 49th Streets. The reason for having both the primary and alternate paths allows for rerouting during periods of traffic congestion.

Based on discussions with the Metropolitan Transit Authority (MTA), a phone system outage would have little effect on subway operations. Commercial phone lines only support communications between supervisory personnel and the information booth attendants. The subway system operations rely on a control and monitoring system which is independent. The tunnel collapse does result in disruption of the tracks connecting the Rector St and South Ferry St stations on the 1 and 9 lines, isolating the South Ferry St station and preventing any passengers from boarding or departing at that location.

The impacts are entered into the model via the GIS interface to the database. In this case, the available supply at substation 15 is reduced to zero as well as portions of eleven feeders as reports of their condition are received. The 1 and 9 line tracks connecting the South Ferry St and Rector St stations also have their capacity reduced to zero as well as the trunk line between the two central offices (note: since the only individual phone lines lost did not impact either the NYSE or the trading offices of concern or have an impact on subway or power system operation, we will not include them in this analysis). The outputs of the model are lists of unmet demands for service - the power and phone outage areas and the number of subway passengers affected (those who can no longer board or depart at the South Ferry St. station). Sufficient redundancy exists in the phone system so that there is no loss of service outside of the area served by the CO at Bowling Green. Demands for power, phone and subway service across the rest of Manhattan are being met.

Affected agencies and businesses start making restoration plans. Power and phone systems would have similar plans consisting of laying temporary shunts along streets, connecting them to intact portions of the original network. Con Ed intends to utilize the reserve capacity from the four substations nearest to the affected area, running shunts from them to the outage area. The model will then aid in determining where individual shunts should be run within the outage area. Similarly, Verizon will run a new trunk line from the Bowling Green / Battery Place central office and tie into the existing line somewhere along Broadway. At the NYC Emergency Operations Center, Con Ed and Verizon are asked to work collaboratively, running lines along the same street sections to the maximum extent possible while each company meets their goal of running the least amount of temporary shunt.

This first restoration plan is formulated similar to the single source fixed charge network flow problem [45]. The objective is to select the smallest set of arcs which meet the flow and capacity constraints. With single commodity systems like power, the set of possible temporary arcs is added to the system's existing arc set and the model will find the new paths through the system. However, in multicommodity systems like phone, the system must first determine the set of temporary arcs that will comprise the shunt. Then the set of paths between each origin and destination must be revised to reflect the installation of the shunt.

The model is used in two steps. First, it must determine where the temporary arcs go. In this scenario, each shunt has a fixed cost, q, which is a function of its length. Since the stated goal was to minimize the amount of shunt used, while also minimizing the number of street sections distributed, a new binary variable, z, is also introduced. Let $z_e^i = 1$ when arc e in infrastructure i has flow on it, $x_e^i > 0$. So, when $z_e^i = 1$, a shunt must be installed at this location and its fixed charge will be incurred. The sum of all the products of z and q represent the total fixed charge and the sum of z represents the number of street sections used. Both of these sums are added to the model's objective function, while a new constraints is added

$$x_e^i \le u_e^i z_e^i \tag{39}$$

After the set of temporary arcs is determined, the new set of paths for the multicommodity system is generated. The model is run again to verify that service is in fact restored.

The proposed solution (called case 1) is shown below and restores power to the affected area in sufficient quantity by using 29 street sections, totaling about two miles of shunts, and restores the phone trunk line using only 8 arcs, i.e. street sections.

Insert Figure 4 here

Upon further review of the Case 1 solution, Con Ed notes that one of the substations selected for this proposed plan is operating at full capacity. This condition is not acceptable and they specify that there must be at least 90 MW reserve capacity. This is added to the model by constraining the slack at the node to be greater than or equal to 90. OEM adds a new requirement that no temporary lines may cross West St. This constraint is added to the model as follows. First, all nodes along West St are transshipment nodes. Therefore, if you sum the installation index, z ,for the temporary arcs which either originate or terminate at nodes in the subset, this sum must be less than or equal to one if no arc crosses West St.

$$\sum z_{j,l}^i + \sum z_{l,v}^i \le 1 \tag{40}$$

for all l in the subset of concern for each infrastructure system.

Adding these constraints (case 2), results in the next solution shown in figure 5.

Insert figure 5 here

This restoration plan uses 34 arcs, i.e. street sections, covering about two and a half miles.

Based on new information from the scene and the advice of other agencies, NYCOEM directs that no lines will be run from West Side substations. Also, because of the heavy equipment that will be involved in restoration and recovery at the tunnel site, the number of power shunts crossing and running along streets will be limited to a capacity of 150 MW. When these constraints are added, the solution results in unmet demands within the outage area. Reducing power shunt capacity to 150MW and not allowing flow from the two West Side substations gives the following solution (case 3) is shown in figure 6.

The model develops the solution of minimum length but the temporary shunts reach capacity before all demands are met. With the slack weighting factor, *k*, for all power nodes equal, shortfalls exist at the most distant points in the power system – power to the normal feeder for subway rectifiers for the subway tracks on the R and W lines between the Rector St and Cortlandt St stations; the feeders to the Bowling Green central office; general outages in some areas west of Broadway.

Insert figure 6 here

This restoration plan involves 50 street sections totaling about three and a third miles.

Now, priorities must be set and tradeoffs must be made. Since this scenario has been focused on the financial sector, restoring phone service and power to them is of primary importance.

Workers are also needed to support restoration of other services in sector; a fully operating subway system is needed (except for the station at South Ferry whose tracks require repair). In order to satisfy the priority of restoring service to the financial sector and making workers available, the slack weighting factors are adjusted to preferentially shift power to the phone and subway systems at the expense of residential and commercial customers. (The remaining area of outage would be of about 39 MW. This amount of demand corresponds to the area highlighted.). This last restoration plan (case 4 and shown in figure 7) utilizes 53 street sections and covers over three and a half miles.

Insert figure 7 here

Con Ed would make the final decisions on which areas remain without service with the advice and consent of NYCOEM. Alternatives could include the use of temporary generators or running shunts from a third East Side substation.

VI. SUMMARY AND SIGNIFICANCE

A. Summary

Models can provide powerful means of understanding [46], monitoring and controlling large-scale infrastructure systems [47]. The need for powerful but parsimonious models is particularly acute as infrastructures increase in complexity, such as when infrastructures are interdependent. The approach taken is to model the interdependent critical infrastructure systems and to provide decision makers with means of manipulating this model for purposes of response and restoration of service. Definitions of various types of infrastructure interdependencies were developed and incorporated into a mathematical representation of interdependent infrastructure systems. This representation permits the development and use of algorithms for identifying solutions to problems associated with disruptions to interdependent critical infrastructures. The models allow representation of infrastructures under conditions of normal operations, post-disruption impact assessment and finally, restoration.

B. Opportunities for Future Work

1) Decision Support System: These models are designed to be imbedded in a decision support system that will employ a database management system for storing data and information on response and restoration resources and have as the human-machine interface, a geographical information system. In its current form, the modeler must change the AMPL code in order to add a constraint. It is envisioned that these operations could be accomplished by additional features, such as drop down menus, which would make the model easier to use by decision makers. Additionally, it is envisioned that this decision support system will have the capability of aiding system designers in increasing the resilience of their systems and increasing their awareness of the effect interdependency plays in the design and operation of these complex systems.

2) Time-Expanded Networks: An approach for time-expanded networks has been conceptualized but must be fully developed. The time-expanded networks will be advanced applications of scheduling problems. Each repair activity, whether it is installation of a temporary component or repair of existing components, is a task which will require resources. Three scenarios for time-expanded networks are foreseen. The existing model is sufficient for the first scenario where all damage and resources are known. It also assumes that all the work proposed can be completed within some acceptable time frame and sufficient resources exist to meet the restoration requirements. The next scenario of the time expanded networks will be based on a fixed set of damage occurring at time t=0, and a set of available resources which vary over time. The time interval t will represent the time needed to complete the shortest task. The objective function will be computed at the end of each time period and the sum of the objective functions will be minimized over all the time periods. It is expected that tasks will be scheduled in the order that has the greatest impact on reducing unmet demand within the resource constraints.

The third case of the time-expanded network will deal with damage which occurs at multiple points in time across the period of analysis. An example of this is a primary shock and aftershocks from an earthquake. When another damage-causing event (such as an aftershock of an earthquake) occurs say at time *t*, it is envisioned that the model would report the set of tasks that were in progress at *t*. The operator will input the new set of damage based on the aftershock and a new set of restoration activities would be proposed. The model, using initial damage reports, would then schedule this new set of tasks along with those tasks partially completed or not yet started.

3) Algorithmic Choices: Solution procedures will be able to take advantage of the structure of the network formulation. Our models have a number of network constraints, which can be exploited to speed up solution both of the linear programming relaxations (through the use of specialized linear programming algorithms, based on either the simplex method [39] or on interior point methods [48]) and of the integer programming problem (through exploitation of the total unimodularity property of network constraint matrices). Algorithms available for solving the integer programming problem include branch-and-cut (see [49, 50] for recent surveys.) and branch-and-price [51]. These approaches use constraint and/or column generation; for larger problems, interior point methods for solving the linear programming relaxations should be considered [52, 53].

C. Conclusion

The anticipated results of the research will improve society's ability to withstand the impact of and respond to events that can disrupt the provision of services that are required for the health, safety and economic well being of the citizenry. Managers of critical infrastructures and emergency response officials will be able to model different event scenarios and assess their impact on the services provided by critical infrastructure systems. With this knowledge, mitigation and preparedness strategies can be formulated and evaluated for their ability to prevent an emergency from escalating into a disaster and, if a disaster does occur, ensure a rapid restoration of critical services.

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APPENDIX

GLOSSARY OF SYMBOLS

b_l^h	The supply or demand at node *l* in infrastructure *h*
b_j^h	The supply or demand at node *j* in infrastructure *h*
b_j^i	The supply or demand at node *j* in infrastructure *i*
b_l^i	The supply or demand at node *l* in infrastructure *i*
c_e^i	The cost associated with flow along arc *e* in infrastructure *i*
C^h	The set of all nodes in *h* upon which depend on any other infrastructure nodes
$C(h,i)$	The set of nodes in h that depend on some other infrastructure *i,*
D^i	The set of all nodes in *i* upon which any other infrastructure nodes depend
$D(i,h)$	The set of nodes in *i* that some other infrastructure h depend upon
e	An arc in set E^i
E^i	The complete set of arcs in infrastructure *i*
$f_{o,p}^i$	The flow on path *p* in O-D pair *o* and infrastructure *i*.
$F(i,h)$	The set of ordered pairs (*j,l*) associated with node *j* in *D*(*i,h*) and node *l* in *C*(*h,i*)
$G(h,i)$	The set of arcs in h that depend on some other infrastructure *i,*
h,i	Infrastructure systems in the set *I*
I	The collection of all infrastructure systems
j,l	A node in set V^i
k_j^i	Weighting factor for node *j* in infrastructure *i*
k_o^i	Weighting factor for O-D pair *o* in infrastructure *i*
$M(i,h)$	The set of ordered pairs (*j,e*) associated with node *j* in *D*(*i,h*) and arc *e* in *G*(*h,i*)
m_o^i	The amount of a commodity or market for O-D pair *o* in infrastructure *i*
n_n	A commodity in set *N*
N	The set of all commodities
o	An origin-destination (O-D) pair in the set

	O^i
O^i	The set of all O-D pairs in infrastructure *i*
p	A path in set P_o
P_o	The set of paths associated with O-D pair *o*
q	General form of a fixed charge
q_e^i	Fixed charge associated with temporary arc *e*
qq_j^i	Fixed charge associated with temporary node *j*
R^i	The subset of arcs which carry multiple commodities
$r_{e,nn}^i$	Variable used in exclusive-or interdependency to indicate if arc *e* is carrying commodity n_n.
s_j^i	The slack associated with node *j* in infrastructure *i*
s_o^i	The slack associated with O-D pair *o* in infrastructure *i*
s_l^h	The slack associated with node *l* in infrastructure *h*
s_j^h	The slack associated with node *j* in infrastructure *h*
s_l^i	The slack associated with node *l* in infrastructure *i*
T^i	The subset of arcs subject to exclusive – or interdependency
u_e^i	The capacity of arc *e* in infrastructure *i*
u_e^h	The capacity of arc *e* in infrastructure *h*
V^h	The complete set of nodes in infrastructure *h*
V^i	The complete set of nodes in infrastructure *i*
$V^{i,+}$	The set of supply nodes in *i*.
$V^{i,=}$	The set of transshipment nodes in *i*.
$V^{i,-}$	The set of demand nodes in *i*.
w	General form of the capacity of a node
w_j^h	The capacity of node *j* in infrastructure *h*
w_m^i	The capacity of node *m* in infrastructure *i*
w_l^h	The capacity of node *l* in infrastructure *h*

w_j^i	The capacity of node *j* in infrastructure *i*
x_e^i	The flow on arc *e* in infrastructure *i*
$x_{e,nn}$	The flow on arc *e* in infrastructure *i* of commodity n_n
x_e^h	The flow on arc *e* in infrastructure *h*
$y_{h,l}^{i,j}$	The connector between node *j* in infrastructure *i* (where it is a demand node) and node *l* in infrastructure *h*
$y_{h,e}^{i,j}$	The connector between node *j* in infrastructure *i* (where it is a demand node) and arc *e* in infrastructure *h*
$y_{h,e}^{i,l}$	The connector between node *l* in infrastructure *i* (where it is a demand node) and arc *e* in infrastructure *h*
y	The general form of the connector variable
z	The general form of the variable indicating when a temporary arc is installed
zz	The general form of the variable indicating when a temporary node is installed
z_e^i	Variable indicating whether or not a temporary arc *e* is to be in infrastructure *i*
$z_{j,k}^i$	Variable indicating whether or not a temporary arc from node *j* to node *k* is to be in infrastructure *i*
$z_{k,l}^i$	Variable indicating whether or not a temporary arc from node *k* to node *l* is to be in infrastructure *i*
$\delta^+(l)$	The set of arcs that enter node *l*
$\delta^-(l)$	The set of arcs that leave node *l*
$\delta(l)$	The set of arcs incident to node *l*

Figure 1 Overview of Manhattan showing area affected by outages.

Figure 2 Close up of affected area showing key locations described in the scenario

Figure 3 Close up of affected area showing subway lines and stations

Figure 4 Restoration plan for case 1 showing locations of power and phone shunts to meet the objective of minimizing the number of shunts installed and the number of street sections used.

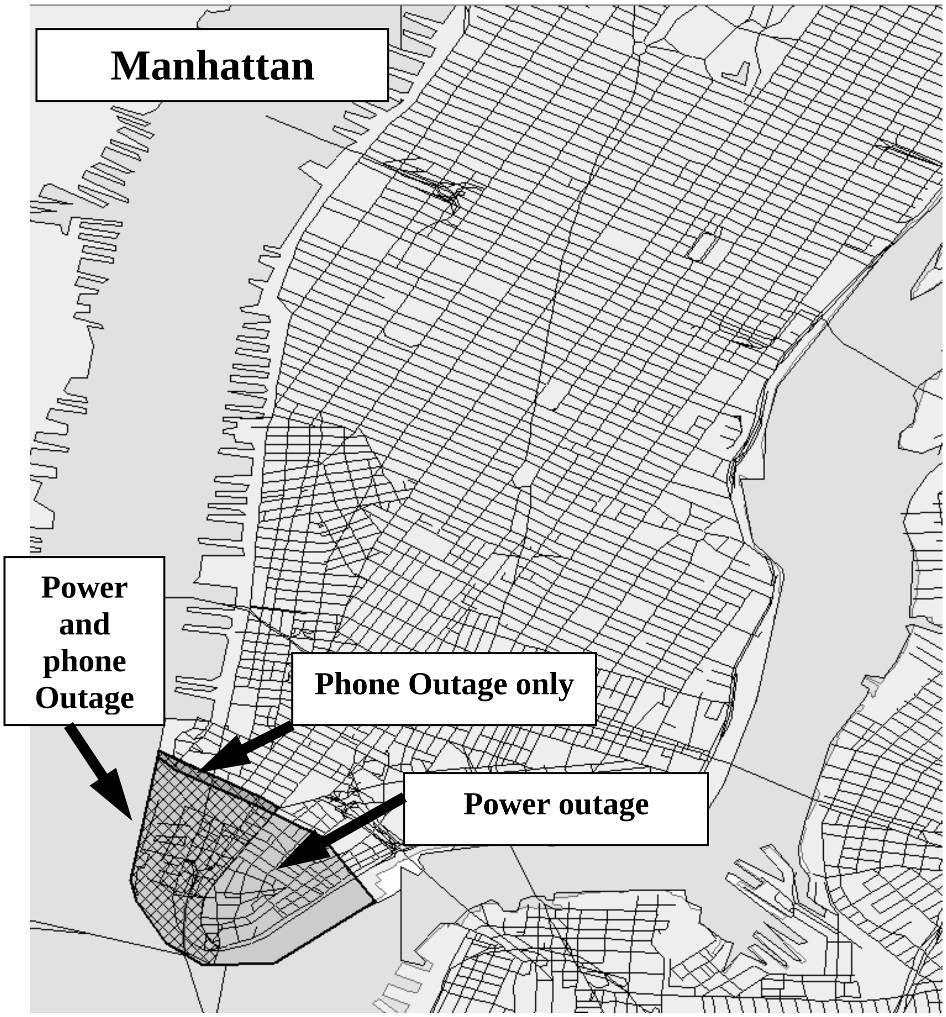
Figure 5 Location of power and phone shunts for case 2 to meeting the new constraint of maintaining 90 MW of reserve capacity at all substations and no shunts crossing West St..

Figure 6 Restoration plan for case 3 meeting the additional requirements of no shunts from west side substations and reducing the capacity of temporary shunts to 150 MW.

Figure 7 Final restoration plan which meets all of the conditions and shows the area where a power outage will still exists. Power is restored to the subway and telecommunications systems.

Footnotes

“Manuscript received……..” The authors are Research Associate, Rensselaer Polytechnic Institute, Troy, NY; Professor of Mathematics, Rensselaer Polytechnic Institute; and Professor, Department of Decision Sciences and Engineering Systems, Rensselaer Polytechnic Institute, Troy, NY. This research has been supported by NSF grants CMS 0139306, Impact of the World Trade Center Attack on Critical Infrastructure Interdependencies; DMII 0228402, Disruptions in Interdependent Infrastructures, A Network Flows Approach and CMS 0301661, Decision Technologies for Managing Critical Infrastructure Interdependencies. The authors also wish to acknowledge the valuable assistance of the Manhattan offices of Consolidated Edison and Verizon, as well as the New York City Office of Emergency Management and the New York State Emergency Management Office.



Manhattan
Power and phone Outage
Phone Outage only
Power outage

Figure 1.

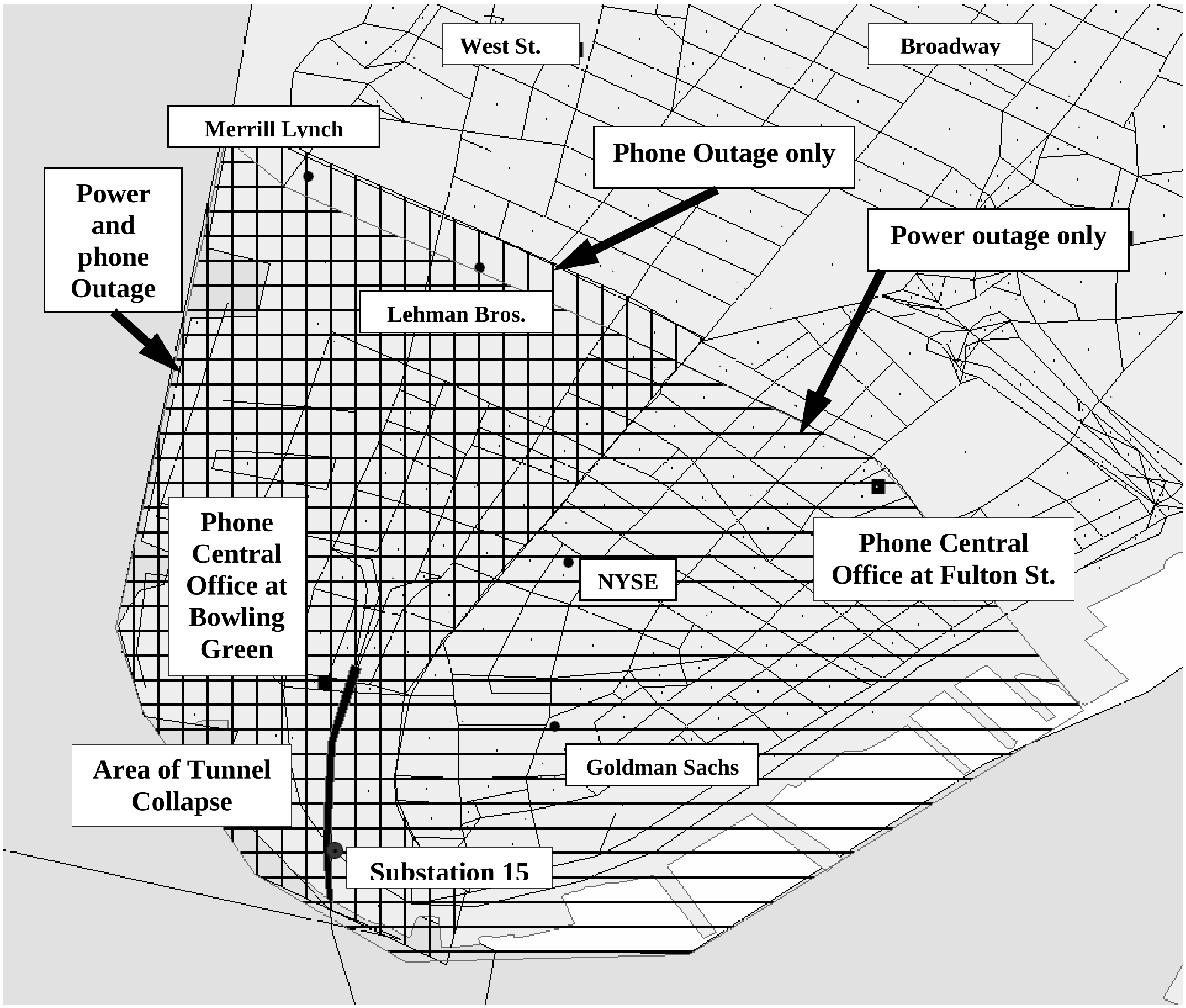

West St.
Broadway
Merrill Lynch
Phone Outage only
Power outage only
Power and phone Outage
Lehman Bros.
Phone Central Office at Bowling Green
NYSE
Phone Central Office at Fulton St.
Area of Tunnel Collapse
Goldman Sachs
Substation 15

Figure 2

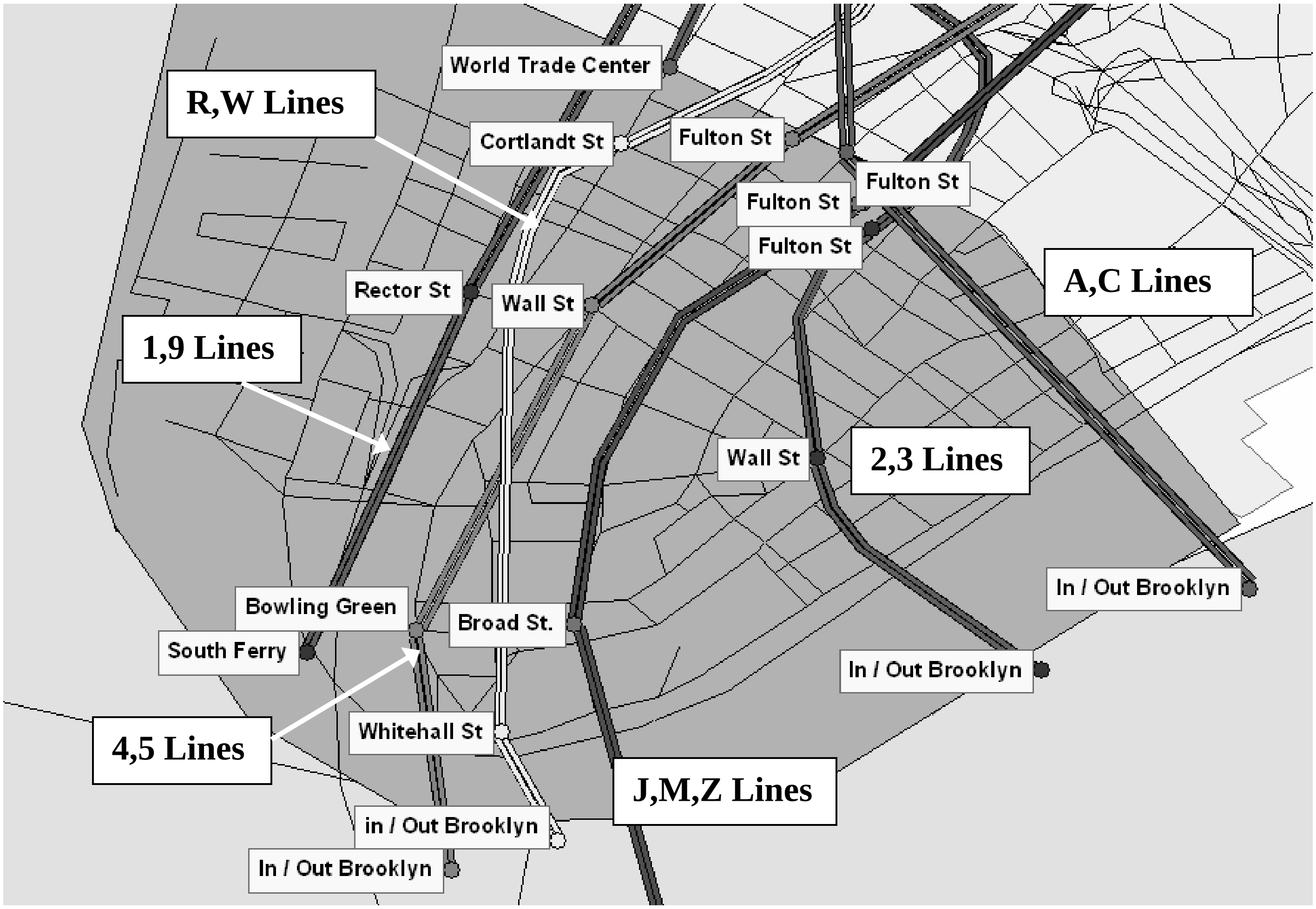

World Trade Center
R,W Lines
Cortlandt St
Fulton St
Fulton St
Fulton St
Fulton St
A,C Lines
Rector St
Wall St
1,9 Lines
Wall St
2,3 Lines
In / Out Brooklyn
Bowling Green
Broad St.
South Ferry
In / Out Brooklyn
4,5 Lines
Whitehall St
J,M,Z Lines
in / Out Brooklyn
In / Out Brooklyn

Figure 3

Figure 4

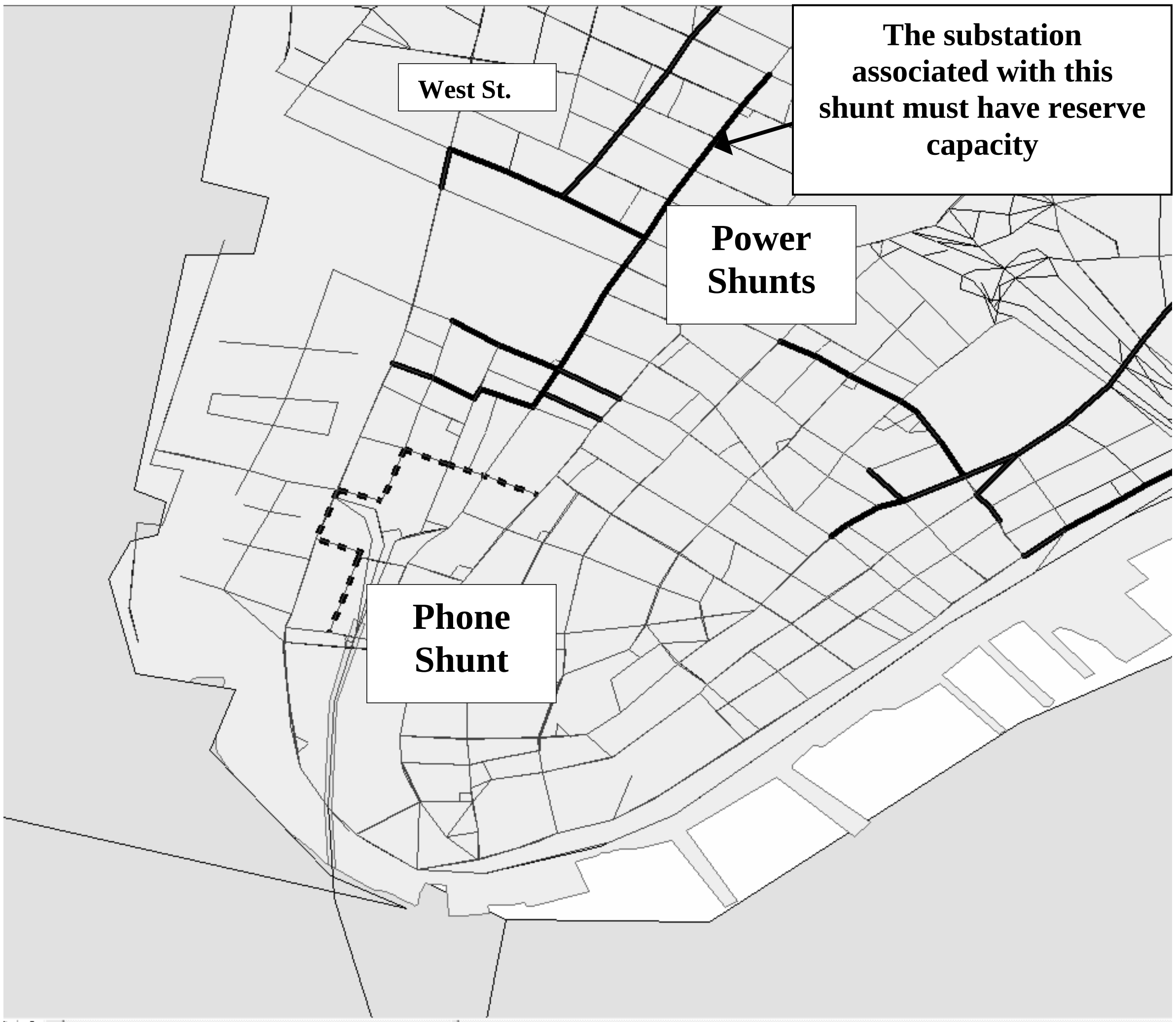

West St.
The substation associated with this shunt must have reserve capacity
Power Shunts
Phone Shunt

Figure 5

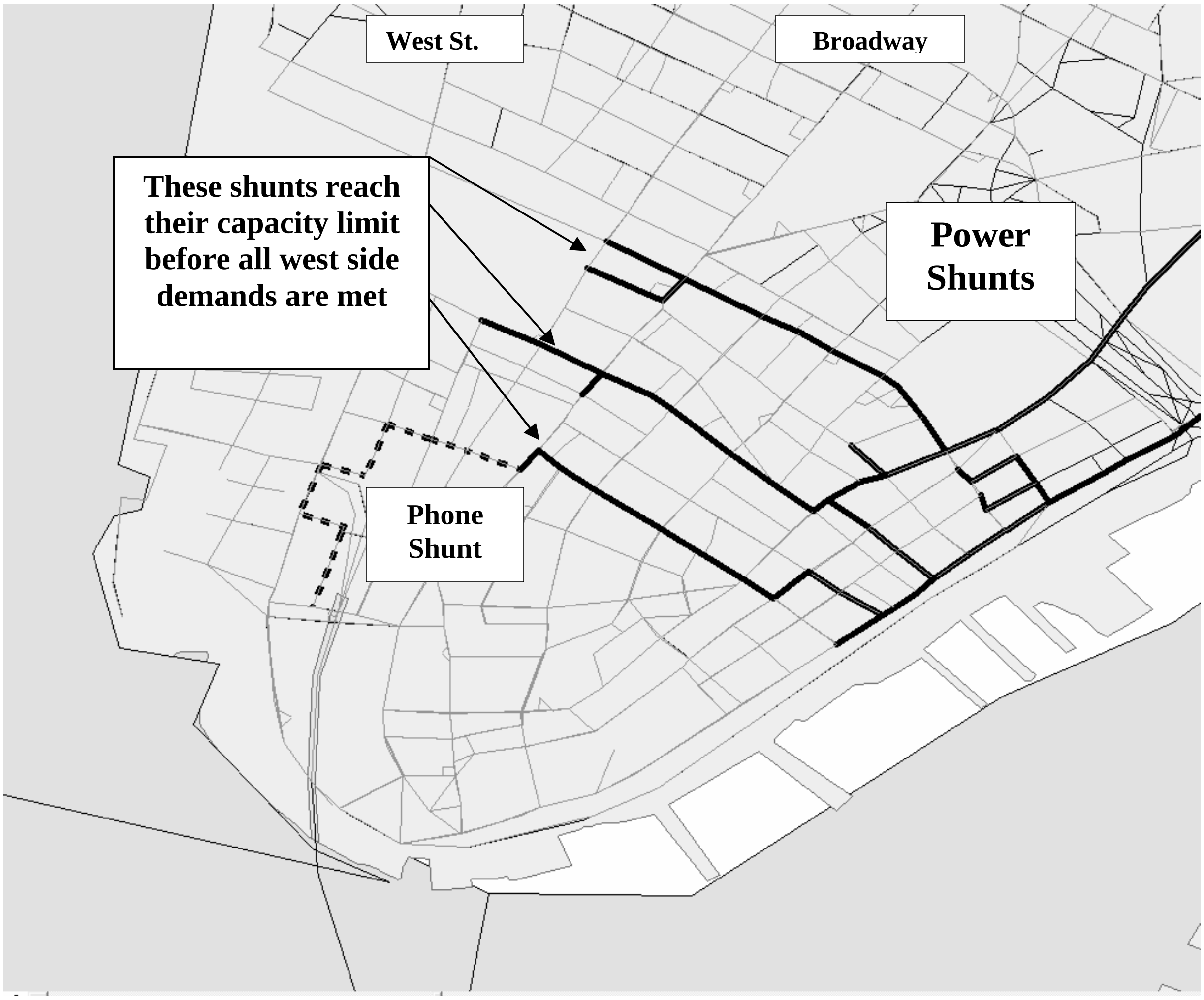

West St.
Broadway
These shunts reach their capacity limit before all west side demands are met
Power Shunts
Phone Shunt

Figure 6

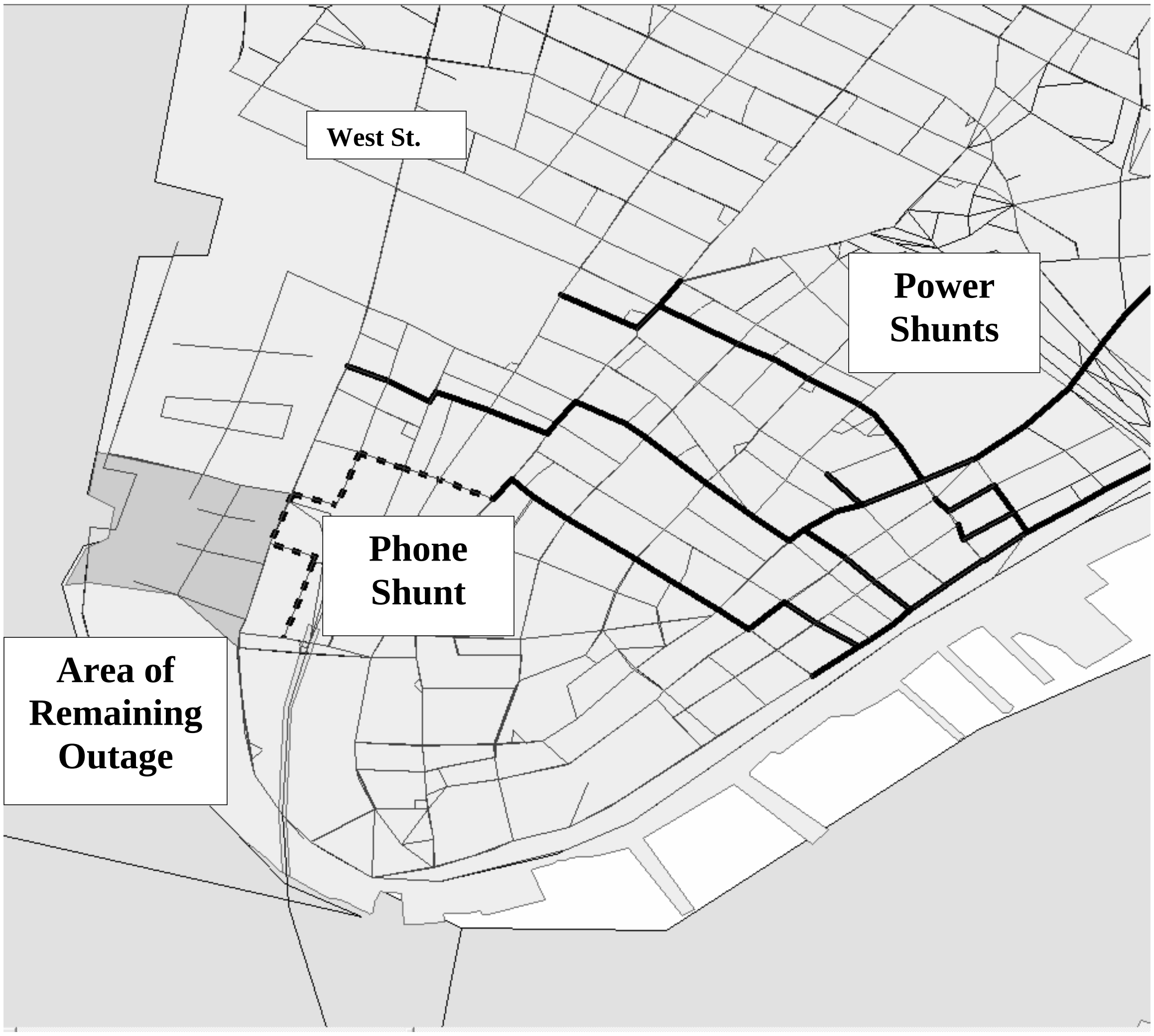


Figure 7