Highlights

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- We design cost curves enabling renewable generators to bid into day-ahead electricity markets
- Proposed market preserves expected profits and cost-to-load equivalence with current practice
- CVaR-based risk adjustment provides transparent trade-off between revenue stability and load costs

Electricity Market Clearing with Renewable Resources Bidding Risk-adjusted Cost Curves

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Abstract

Renewable power generators typically participate as price-takers in current electricity markets, and market clearing prices are determined by cost curves of conventional generators while treating renewable generation as negative load. In this paper, we analyze a new design of electricity markets where renewable power generators bid their *risk-adjusted* cost curves into the market, just like the cost curves of conventional generators. We establish that on average, the total revenue of both conventional and renewable generators, as well as the total cost to consumers, under the proposed market is the same as that in current practice. However, compared to current practice, the variability in the total consumer cost is always lower in the proposed market, i.e., the risk of price variability due to uncertainty in renewable generation is not borne by the consumers. Further, under certain operating conditions, the proposed market also results in lower variability in the revenue earned by the renewable generators. We then show how these cost curves should be adjusted to incorporate the risk of shortfall into the bidding and unit commitment process through the Conditional Value-at-Risk (CVaR) metric. Using data from the Texas grid, we evaluate the bid curves for renewables and quantify the tradeoff between renewable revenue and cost-to-load. We also study how CVaR-based adjustments for shortfall risk impact allocations and payments at market clearing. While equivalent in terms of average cost and revenue measures, the new market design insulates customers from the price risk associated with the uncertainty of renewable power generation. Further, it allows renewables to be direct participants in the bidding process and bid according to their risk-return tolerance level.

1. Introduction

Investment into renewable energy assets has been growing exponentially in recent years. Government subsidies on renewable energy generators and other equipment aimed at aiding energy transitioning are also providing strong incentives in this direction (Renews.biz; PV Magazine). In the United States, state-wide policy initiatives in New York have aimed to deliver 70% of its energy from renewable energy resources by 2030 and 100% emissions-free electricity supply by 2040 (New York ISO, 2021), utilizing 3 GW of energy storage and 9 GW of offshore wind power.

Despite this surge in renewable energy penetration, the majority of renewable generators today act as *price-takers* in the electricity market, that is, they are either not allowed or unwilling to bid into the market like conventional generators. This appears justifiable on first thought, since: (a) the operating costs for renewable generation are nearly zero, unlike conventional generators who have to pay for fuel and bid their generation cost curves into the market; (b) renewable generation is associated with a high degree of uncertainty, making it difficult to provide guaranteed day-ahead commitment like conventional generators. Renewable generators, therefore, mostly supply energy in real-time, and are paid at market clearing prices that are determined from the cost curve bids of the conventional generators. In running the unit commitment and economic dispatch problems for the day ahead market (DAM) and the real-time market (RTM), the system operator can treat the renewables as "negative load".

Since the renewable production is stochastic and not fully predictable, stochastic variants of the unit commitment problem need to be considered. This has led to a vast amount of work on stochastic unit commitment problems (Conejo, Carrión and Morales, 2010; Zheng, Wang and Liu, 2015; Håberg, 2019), which attempt to pose and solve this problem through multi-stage stochastic optimization, chance-constrained programming, robust optimization, stochastic dynamic programming, and other related approaches. Recent works such as Hogan (2022); Leslie, Stern, Shanker and Hogan (2020); Frew, Clark, Bloom and Milligan (2017); Ela, Milligan, Bloom, Botterud, Townsend, Levin and Frew (2016); Blazquez, Fuentes-Bracamontes, Bollino and Nezamuddin (2018); Bienstock, Dvorkin, Guo, Mieth and Wang (2024) either directly assume renewables to be price-takers, or that renewables will be bidding their actual cost (which is close to zero) effectively implying the same. Such a market structure implies that even as renewable supply increases to meet a bulk of the energy demand, market prices are going to be determined by a few conventional generators which can provide nearly risk-free energy commitments. This is undesirable from an economic perspective, as it does not incentivize renewables to innovate, or reduce their supply variability – the root cause behind their inability to bid into the market – by investing in storage. Wind generators

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are increasingly investing in co-located storage (Dattaray, Wall and Terzija, 2021), which could allow them to commit more reliably to an energy supply, much like conventional generators. Further, as envisioned in our recent work (Osipov, Naqvi, Palepu, Onyeziju, Kirk-Davidoff and Du, 2026), a renewable generator can segment its supply and offer a fraction of its forecast energy as a highly reliable contract, comparable to that of conventional generators. Without a mechanism to bid into the market while adjusting for risk, such technological innovations would not happen, and much of the long-term benefits of high renewable penetration will be forfeited.

With this motivation, this paper analyzes a market in which renewable generators can bid into the market, just like conventional generators. We make several contributions, as listed below. First, we derive expressions for the *expectation-adjusted* cost curves that the renewables can bid into the market. Through theoretical analysis, we show that the expected profit earned by both conventional and renewable generators and the cost-to-loads are similar between our proposed market and an idealized baseline representation of the current market. Next we derive what this renewable cost curve should be to maximize its expected profit, and compare it with that derived earlier for cost optimality. Afterward, we compare the price risk borne by the conventional generators, renewable generators, and consumers between two market models by determining the variance of generator profits and cost-to-load. The proposed market could either increase or decrease the risk exposure for renewable generators, depending on the operating conditions. However, a key feature of the new proposed market is that it *consistently lowers the price risk for consumers* compared to current practice. In other words, unlike current practice, the price risk associated with the uncertainty (unpredictability) of renewable generation is not passed on to consumers in the proposed market.

In markets with high renewable penetration, downside risk arises when real-time output falls short of day-ahead commitments, exposing renewable bidders to substantial RTM charges. To address this vulnerability, we incorporate the Conditional Value-at-Risk (CVaR) concept into renewable bid curves as a risk-aware adjustment (Rockafellar and Uryasev, 2000). CVaR captures expected losses in adverse scenarios, weighted by a risk-aversion parameter, while preserving convexity and maintaining piecewise linearity under finite scenarios. At the market level, this adjustment steepens renewable offers, moderates DAM commitments, and improves downside protection as a transparent trade-off in expected cost-to-load.

Finally, we present a numerical study of the new market in terms of the above metrics using a Texas load and generation data set, accounting for forecast errors in renewable generation rates. The numerical results corroborate our theoretical findings. In particular, they show that on average the generator profits and cost-to-load remain comparable between the two markets even under renewable generation forecast errors, and the variability in cost-to-load is consistently lower in the proposed market. Our evaluation also demonstrates how varying risk-aversion weights affects renewable commitments, profits, and load costs, thereby quantifying the cost–risk trade-offs inherent in the proposed market design.

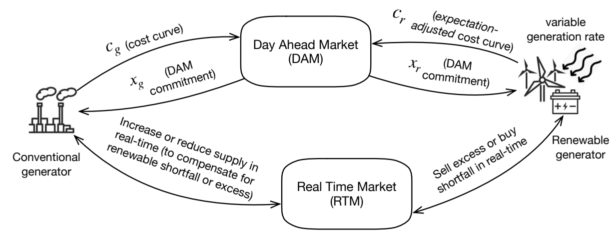


Figure 1: High-level overview of the market with both conventional and renewable bidders.

This paper is structured as follows. Section 2 reviews related work on renewable bidding and electricity market design. Section 3 describes the system model and the simplifying assumptions made toward analytical tractability. Section 4 computes analytical expressions for the renewable cost curves and payments made to both conventional and renewable generators on market clearing. Section 5 utilizes those expressions to establish the equivalence of the proposed market with current practice in terms of expected social utility, generator profit and load cost. Section 6 analyzes an optimal linear bid for renewables and its revenue versus cost-to-load trade-off. Section 7 compares the variability of generator profits and cost-to-load across the two market models. Section 8 develops risk-adjusted cost curves based on CVaR and characterizes their implications for allocations and payments. Section 9 discusses the results of our numerical study on the ERCOT Texas grid data (Electricity Reliability Council of Texas, 2024c). Section 10 concludes by listing future research directions.

2. Related Work

Whether and how renewable generators should transition from passive price-takers to active market participants has attracted some attention in recent years. Kenis, Höschle and Bruninx (2022) derive closed-form best-response functions for strategic quantity bids by wind producers acting as price-makers in the day-ahead market (DAM). Johanndeiter and Bertsch (2024) provide empirical evidence from the Iberian market that a subset of solar operators submit price bids above zero, despite their near-zero marginal costs, consistent with arbitrage incentives across sequential auctions. Both works assume or cite active market participation by renewables but do not theoretically analyze market outcomes resulting from such participation in comparison with current market practice.

Several studies address bid curve design from the perspective of stochastic generation assets. Silva-Rodriguez, Sanjab, Fumagalli and Gibescu (2025) propose a light-robust DAM formulation with uncertainty bids, showing that, under single-pricing imbalance settlement, incentives to deviate from truthful uncertainty ranges can arise when positive system imbalances are anticipated. Gupta and Palepu (2024) construct risk-free tranche bid curves through contingent-claim pricing under a risk-neutral valuation framework, while Herding, Ross, Jones, Endler, Charitopoulos and Papageorgiou (2024) obtain bidding curves numerically via a multi-objective stochastic MILP that jointly minimizes expected cost and CVaR for a specific microgrid. Carrión, Domínguez and Oggioni (2025) study optimal participation of a price-taking wind producer in a hybrid intraday market, using a multi-stage stochastic program with CVaR to trade off expected profit and downside risk with a focus on the producer-side trading across sessions. Closer to a system perspective, Zhao, Delikaraoglou, Dvorkin, Lamadrid and Botterud (2025) optimize variable renewable energy bidding curves in a two-settlement market via a bilevel framework that anticipates real-time redispatch under scenario uncertainty. They show that, under the assumption of zero marginal cost for renewables, a single zero-price segment with optimized quantities can achieve the same expected system cost as a richer price, quantity formulation and approximate stochastic joint dispatch. In contrast to these prior approaches, we derive closed-form cost curves directly from the existing two-stage DAM–RTM clearing and prove their market-equivalence to the negative-load benchmark. We also analyze price risk of market participants (generators as well as loads) in the proposed market, which is not done in prior work.

Our use of CVaR for incorporating risk-sensitivity to the bids is broadly related to risk management in renewable trading that has been studied at the forecasting stage (Wang, Zhou, Zhang, Lin and Wang, 2024) and through coordinated multi-market bidding (Miskiw, Kraft and Fleten, 2025). Unlike prior work, however, we embed CVaR directly into closed-form analytical renewable bid curve construction, yielding a risk-adjusted offer that steepens with the risk-aversion parameter while preserving the piecewise-linear convex structure required for market submission.

3. System Model

This section provides a description of the system model, including the underlying assumptions, which will be used to define the new market with renewable bidders and compare it with the current practice. We make several simplifying assumptions and approximations on electricity market operations to keep our analysis tractable.

We consider the unit commitment (UC) problem in the electricity market for an entire day, solved at an hourly time granularity. We ignore factors that couple the UC problems across different hours, such as ramping constraints of conventional generators and storage capacities that may be associated with renewable generators. This allows us to treat the UC problem in each hour separately from the others. As in traditional electricity markets, we assume that UC happens in two time stages: day-ahead and real-time. However, in the market we analyze, we assume renewable generators bid (expectation-adjusted) cost curves, just like conventional generators. Thus, in the first stage, which corresponds to day-ahead market (DAM) clearing, based on the day-ahead forecast demand for any given hour and the bid (cost) curves submitted by the conventional and renewable generators, the generators are committed to provide a certain amount of electricity in that hour. In the second stage, which corresponds to real-time market (RTM) clearing, the actual renewable energy supply and the demand are realized, and renewable generators who have less supply than committed must make up by buying the shortfall from the RTM. This shortfall must be obtained from other renewables with excess supply, or conventionals which sell additional supply in the RTM to meet this real-time shortfall. We assume that renewable energy is not curtailed; therefore, if there is an excess of supply across all renewables, the conventionals reduce their supply in the RTM. We also assume marginal cost pricing, i.e., market participants have to buy or sell energy at the marginal cost of power production, determined from the submitted cost curves of the generators that are committed at market clearing.

Fig. 1 provides an illustration of the day-ahead and real-time market operations that we analyze, assuming a single renewable generator unit. More specifically, for the hour under consideration, we make the following assumptions:

Assumption 1. *The stochastic demand, d, is always greater than the maximum possible power available from the renewable generators.*

Assumption 2. *The cost of conventional generators is aggregated into a single cost function, $c_g(x_g)$, which is monotonically increasing and convex in x_g. However, the cost curves of the renewable generators ($c_r(x_r)$ for renewable r) must be considered individually and will be analytically derived in Section 4.*

Assumption 3. *The demand and renewable power generation realized in real-time are modeled using scenarios $\xi^k \in \Xi$.*

Assumption 4. *The set of conventional generators is willing to reduce their production to accommodate excess generation by renewable generators. In addition, the conventional generators can increase generation to cover any shortfall in renewable generation.*

Assumption 5. *If a renewable r's real-time generation, $\lambda_r(\xi^k)$, differs from its DAM commitment, x_r, then it can either purchase or sell energy in RTM at the prevailing market price, ensuring that its DAM commitment is met and none of its produced power is curtailed.*

Assumption 6. *The cost function $c_j(x_j)$ has no fixed costs for any generator j. The justification for this is that our model is a single-period model, so implicitly we are assuming that any start-up costs are not applicable. Mathematically, this is ensured by adding a constant k_j to any calculated cost function so that $c_j(0) = 0$.*

Following Assumption 1, let $\bar{d} := \mathbb{E}_{\Xi}[d]$ denote the day-ahead forecast (or expected) demand used in DAM clearing.

Note that Assumption 2 follows from the convexity and monotonicity of the cost curve of each conventional generator. When there are many such conventional generators, their bids can be "stacked" to form a monotonically increasing cost function, which can be approximated by a quadratic or cubic cost curve. Assumption 4 implies full liquidity of the conventional generator, which ensures that all renewable power produced is fully utilized (i.e., not curtailed). Under these assumptions, the ISO can solve an optimization problem to find the allocations of the conventional and the renewable generators in the DAM. Depending on the particular scenario ξ^k arising, the generators can trade among themselves in the RTM at the market price. The RTM price is determined by the total renewable production realized in real-time, and the cost curve of the conventional generators. Also note that some of the assumptions stated above are driven by the need to keep the model tractable for mathematical analysis, and to derive closed-form expressions for the renewable cost curves, system cost, and generator profit measures. The consideration of multi-period models and incorporating ramping, startup, and shutdown constraints are deferred to future work.

4. Analytical Expressions

In this section, we derive analytical expressions of the cost curves of each renewable generator, the market clearing payments, and profits of each type of generator in both DAM and RTM.

4.1. Calculating a Cost Curve for Each Renewable

To submit a bid curve, a renewable generator calculates its expected cost function for each accepted bid value. Let R denote the set of renewable generators and x_g denote the power generated by the aggregated conventional generators, with corresponding cost curve $c_g(x_g)$ with derivative $c'_g(x_g)$.

For each scenario ξ^k, let the total renewable output be denoted by

$$\lambda^{TOT}(\xi^k) = \sum_{r \in R} \lambda_r(\xi^k). \tag{1}$$

Under Assumptions 1 and 4, total conventional production x_g in the DAM and RTM for scenario k satisfies

$$x_g^{DAM} + x_g^{RTM} = (d - \lambda^{TOT})(\xi^k). \tag{2}$$

For a given scenario ξ^k and a given renewable generator r with accepted DAM bid of x_r^{DAM}, exploiting Assumptions 4 and 5, the RTM cost for r is:

$$\check{c}_r\left(x_r^{DAM}, \xi^k\right) = c'_g\left((d - \lambda^{TOT})(\xi^k)\right)\left(x_r^{DAM} - \lambda_r(\xi^k)\right). \tag{3}$$

Proposition 4.1. *Define the parameters*

$$\alpha \;:=\; \mathbb{E}_\Xi\left[c'_g\left((d - \lambda^{TOT})(\xi^k)\right)\right] \tag{4}$$

$$\beta_r \;:=\; \mathbb{E}_\Xi\left[c'_g\left((d - \lambda^{TOT})(\xi^k)\right)\lambda_r(\xi^k)\right], \quad \forall r \in R. \tag{5}$$

Then the expected RTM cost for the renewable r is

$$\check{c}_r(x_r^{DAM}) = \alpha x_r^{DAM} - \beta_r. \tag{6}$$

From Assumption 6, we use the adjusted cost function

$$c_r(x_r) = \alpha x_r. \tag{7}$$

We note that the cost curves $\check{c}_r$ in (6) are linear in nature, and so are the adjusted cost curves c_r in (7). The parameters α and β_r can be interpreted as the expected RTM price and the expected revenue of renewable r in the RTM, respectively. Here, α is independent of r, while the term β_r varies with each renewable, and depends on the power produced in that hour by renewable r. As we will see in Section 4.2, dropping the constant terms β_r from the cost curves only changes the objective function of DAM clearing by a constant, and therefore does not impact the allocations. However, the term β_r does impact the revenue earned by the renewable, as we show in Section 4.3.

Calculating α requires estimation of the price statistic $c'_g\left((d - \lambda^{TOT})(\xi^k)\right)$ for that hour, which can be made from study of historical prices under a similar weather pattern. It can be calculated by the ISO–which typically utilizes third party services for such forecasts–and communicated to the renewables. Alternatively, α can be obtained separately by each renewable (along with its individual revenue estimate, β_r) by utilizing similar third party analytics and forecasting services. In either case, the ISO uses a single, system-wide α for market clearing, since α enters every renewable's cost curve $c_r(x_r) = \alpha x_r$ in (8). The parameter β_r, by contrast, is renewable-specific—it depends on the individual output $\lambda_r(\xi^k)$ and is naturally computed by each renewable using its own generation forecast, independently of the ISO. The equivalence results of Section 5 hold exactly when $\alpha = \mathbb{E}_\Xi[c'_g\left((d - \lambda^{TOT})(\xi^k)\right)]$, and approximately when α is estimated from historical data with small error: the deviation from exact equivalence in expected profits is proportional to the estimation error, and diminishes as the calibration dataset grows more representative.

Finally, note that the cost curves $\check{c}_r$ and c_r are valid for $0 \le x_r^{DAM} \le \max_{\xi^k \in \Xi} \lambda_r(\xi^k)$ for each $r \in R$. Calculation of α and β_r using realistic data from the Texas grid is illustrated in Appendix A.

4.2. Solving the ISO problem with these cost curves

Given the cost curves, the ISO solves the DAM problem:

$$\begin{aligned} \min_{x_g^{DAM}, \{x_r^{DAM}\}} \quad & c_g(x_g^{DAM}) + \sum_{r \in R} c_r(x_r^{DAM}) \\ \text{subject to} \quad & x_g^{DAM} + \sum_{r \in R} x_r^{DAM} = \bar{d}, \\ & x_g^{DAM} \ge 0, \quad 0 \le x_r^{DAM} \le \max_{\xi^k \in \Xi} \lambda_r(\xi^k), \forall r \in R. \end{aligned} \tag{8}$$

Substituting for $c_r(x_r^{DAM})$ from (7) and then using the first constraint to eliminate $\sum_{r \in R} x_r^{DAM}$ gives a convex problem in the single variable x_g^{DAM}. We let $\lambda_{\max}^{TOT} = \max_{\xi^k\in\Xi}\sum_{r\in R}\lambda_r(\xi^k)$ and $\lambda_{\min}^{TOT} = \min_{\xi^k\in\Xi}\sum_{r\in R}\lambda_r(\xi^k)$ denote the maximum and minimum possible production from the renewables, respectively. The bounds on x_r^{DAM} imply $\bar{d} - \lambda_{\max}^{TOT} \le x_g^{DAM} \le \bar{d}$.

Hence the set of solutions x_g^{DAM} to (8) is

$$x_g^{DAM*} \begin{cases} = \bar{d} - \lambda_{\max}^{TOT} \text{ if } c_g'(\bar{d} - \lambda_{\max}^{TOT}) > \alpha \\ = \bar{d} \text{ if } c_g'(\bar{d}) < \alpha \\ t \in \{[\bar{d} - \lambda_{\max}^{TOT}, \bar{d}] \,:\, c_g'(t) = \alpha\} \text{ otherwise.} \end{cases} \tag{9}$$

Once the ISO has solved for x_g^{DAM*}, it can then choose any assignment $\{x_r^{DAM} \,:\, r \in R\}$ satisfying (8).

The mechanism through which estimation errors propagate is through (9): an overestimate of α increases x_g^{DAM}, committing more conventional generation and less renewable in the DAM, while an underestimate has the opposite effect. RTM settlement prices, being determined by actual real-time output, are independent of the α estimate and thus unaffected.

4.3. Who gets paid when?

All the RTM payments are made at the marginal price $c_g'\big((d - \lambda^{TOT})(\xi^k)\big)$, comprising adjustments to production schedules due to the realization of a particular scenario ξ^k.

The DAM payments would use the marginal price in the solution of (8), namely α. Thus, we would have payments of

- αx_g^{DAM*} to the conventional generators,
- αx_r^{DAM*} to each renewable generator, where x_r^{DAM*} is the optimal DAM allocation of renewable r.

Thus, each generator's DAM payment would be equal to the product of its assigned production value and its marginal cost. The total payout by the ISO would be

$$\text{cost-to-load} = \alpha\bar{d} + c_g'\big((d - \lambda^{TOT})(\xi^k)\big)(d - \bar{d}). \tag{10}$$

With this payout, the expected DAM+RTM profit for each renewable generator would be

$$\text{profit}_r = \alpha x_r^{DAM*} - (\alpha x_r^{DAM*} - \beta_r) = \beta_r. \tag{11}$$

From (5) and the convexity of c_g, we observe that this profit will be high when λ_r is negatively correlated with λ^{TOT}, as we intuitively expect.

In scenario ξ^k, the conventional generators would produce $d - \lambda^{TOT}(\xi^k)$, so the net profit for the conventional generators would be

$$\begin{aligned} \text{profit}_g(\xi^k) &= \alpha x_g^{DAM*} - c_g((d - \lambda^{TOT})(\xi^k)) \\ &+ c_g'\big((d - \lambda^{TOT})(\xi^k)\big)((d - \lambda^{TOT})(\xi^k) - x_g^{DAM*}), \end{aligned} \tag{12}$$

where the last term is the net revenue received from the renewable generators in the RTM. (The net revenue received in RTM is negative if there is a renewable surplus with $x_g^{DAM*} > (d - \lambda^{TOT})(\xi^k)$.)

If the conventional generators were to produce exactly x_g^{DAM*} in the RTM, their profit would be

$$\text{profit}_g^{DAM} = \alpha x_g^{DAM*} - c_g(x_g^{DAM*}). \tag{13}$$

The following lemma states that the profit of the conventional generator increases if it adjusts its production so all generated renewable energy is consumed. The proof of the lemma (Appendix B) exploits the convexity of $c_g(x_g^{DAM})$, in particular the subgradient inequality.

Lemma 4.2. *Assume we have a scenario with renewable shortfall or surplus, so $\lambda^{TOT}(\xi^k) \neq d - x_g^{DAM*}$. Then the conventional generators make a greater profit by producing $(d - \lambda^{TOT})(\xi^k)$ in the RTM than by producing x_g^{DAM*}.*

In particular, consider the case that the conventional generators back off, so their production decreases by $\Delta = x_g^{DAM*} - (d - \lambda^{TOT})(\xi^k)$. Their DAM revenue is $c_g'(x_g^{DAM})x_g^{DAM}$, their costs are $c_g(x_g^{DAM} - \Delta)$, and they also have to buy Δ from the renewables at the market price of $c_g'(x_g^{DAM} - \Delta)$. Because their costs decrease and they are buying from the renewables in the RTM at a lower price than they are selling to the ISO in the DAM, their profit increases.

5. Markets Equivalence

In this section, using the analytical expressions derived in Section 4 for the market with renewable bidders, we establish its equivalence with a baseline model where renewables are only price takers. In the baseline, renewable energy sources bid into the DAM their forecasted energy at zero price for each hour of the next day. Conventional generators also submit their cost curves into the DAM, and the ISO clears the market by minimizing the total system cost against loads. In real time, renewable resources will supply their available energy and settle any excess or deficit amount based on the real-time locational marginal prices. This baseline market clearing formulation is similar to the two-stage stochastic unit commitment problem considered in the literature (Zheng et al., 2015; Håberg, 2019). The baseline model is not a practical approach, as renewable resources normally do not submit the full amount of forecasted energy in the DAM, as there is risk in energy shortfall in real time. For the loads, this model reduces the energy prices and the cost-to-loads to ideally the lowest level in the DAM. The next few subsections compare this baseline with the proposed market with renewables bidding cost curves as defined in Section 4.1, in terms of profit and revenue of both conventional and renewable generators, and cost-to-load.

5.1. Profit and Revenue

We make the following assumption for the rest of Section 5.

Assumption 7. *Assume the aggregated cost function of the conventional generators is quadratic, i.e., $c_g(x_g) = ax_g^2 + bx_g + c$ with $a, b,$ and c being some constant parameters.*[1]

The use of quadratic cost curves for conventional generation is quite standard (Wood, Wollenberg and Sheblé, 2013). Derivation of some of the results of this section exploits this quadratic nature of the cost function. Extension of our results to more general cost curves is left for future work. As a consequence of Assumption 7, the gradient $c_g'(x_g^{DAM})$ is a linear function of x_g^{DAM}.

Consequently, $\alpha := \mathbb{E}_\Xi(c_g'(d - \lambda^{TOT})) = c_g'(\bar{d} - \mathbb{E}_\Xi(\lambda^{TOT}))$. We denote $\bar{\lambda} := \mathbb{E}_\Xi(\lambda^{TOT})$. In the baseline model, $x_g^{DAM} = \bar{d} - \bar{\lambda}$, so this x_g satisfies the optimality condition (9) for the proposed model, in particular $\bar{d} - \lambda_{\max}^{TOT} < x_g^{DAM} < \bar{d}$. Hence, we have the following proposition (proof in Appendix C).

Proposition 5.1. *Assume Assumptions 1–7 hold. The optimal x_g^{DAM} in the proposed model (8) is the same as the optimal x_g^{DAM} in the baseline model.*

The equivalence of x_g^{DAM} in the two markets is required for the subsequent results in Section 5.

Both the baseline and proposed models allow for surplus renewable generation, with the conventional generators backing off as necessary. Hence the optimal choices for x_g in the real-time market are also identical. Since the DAM and RTM values of the optimal x_g^{RTM} are identical, and since the payments to the conventional generators are determined by the marginal costs c_g', the profits of the conventional generators in any scenario are the same in the proposed and baseline models. However, the proposed model is practical as the nonzero cost curves provide some price protections for the renewables against variations in real-time energy production. The proposed model also readily extends to risk arbitrage, as shown in Section 7.

We now analyze the corresponding total profits, i.e., profit = profit^{DAM} + profit^{RTM} for the renewable generators in each market. In any scenario (ξ) in the baseline market, all electricity from the renewable generators is sold in the RTM, so we have

$$\text{profit}_{r_{TOT}}^{\text{baseline}}(\xi) = c_g'((d - \lambda^{TOT})(\xi))\lambda^{TOT}(\xi). \qquad (14)$$

In the proposed new market:

$$\begin{aligned} &\text{profit}_{r_{TOT}}^{\text{proposed}}(\xi) \\ &= \alpha x_{r_{TOT}}^{DAM^*} + c_g'\big((d-\lambda^{TOT})(\xi)\big)(\lambda^{TOT}(\xi) - x_{r_{TOT}}^{DAM^*}) \\ &= \text{profit}_{r_{TOT}}^{\text{baseline}}(\xi) + (\alpha - c_g'\big((d-\lambda^{TOT})(\xi)\big))x_{r_{TOT}}^{DAM^*}. \end{aligned} \qquad (15)$$

Hence, in any scenario ξ, we have

$$\begin{aligned} &\text{profit}_{r_{TOT}}^{\text{proposed}}(\xi) - \text{profit}_{r_{TOT}}^{\text{baseline}}(\xi) \\ &= \big(\alpha - c_g'\big((d - \lambda^{TOT})(\xi)\big)\big)x_{r_{TOT}}^{DAM^*}. \end{aligned} \qquad (16)$$

[1]Under normal operations, the higher polynomial terms can be neglected because the conventional generators are not dispatched beyond their efficient generation zones.

Using (4), the expected difference in profit between the two models is $\big(\alpha - \mathbb{E}_\Xi\big[c_g'\big((d - \lambda^{TOT})(\xi)\big)\big]\big)x_{r_{TOT}}^{DAM^*} = 0$. In summary, we have the following proposition.

Proposition 5.2. *Assume Assumptions 1–7 hold. The expected profits of both the conventional and renewable generators in the proposed market are equivalent to their corresponding values in the baseline market.*

5.2. Cost-to-Load Analysis

We define cost-to-load as the summation of the profit of the conventional generators, the profit of the renewable generators and the cost for the conventional generators to produce their energy, i.e., CTL = profit$_g$ + profit$_{r_{TOT}}$ + GenCost$_g$. Equivalently, CTL = revenue$_g$ + revenue$_{r_{TOT}}$. This is what the ISO pays in total under a particular scenario. We obtain:

- With the proposed model, cost-to-load is

$$\text{CTL}^{\text{proposed}}(\xi) = \alpha\bar{d} + c_g'((d - \lambda^{TOT})(\xi))(d(\xi) - \bar{d}). \qquad (17)$$

- For any scenario ξ, exploiting the equality $d = x_g^{DAM} + x_g^{RTM} + \lambda^{TOT}$, the baseline cost-to-load is

$$\begin{aligned} &\text{CTL}^{\text{baseline}}(\xi) \\ &= \text{profit}_g^{\text{baseline}}(\xi) + c_g((d-\lambda^{TOT})(\xi)) + \text{profit}_{r_{TOT}}^{\text{baseline}}(\xi) \\ &= \alpha x_g^{DAM^*} + c_g'((d-\lambda^{TOT})(\xi))(x_g^{RTM}(\xi) + \lambda^{TOT}(\xi)) \\ &= \alpha x_g^{DAM^*} + c_g'((d-\lambda^{TOT})(\xi))(d - x_g^{DAM^*}), \end{aligned} \qquad (18)$$

In a shortfall scenario ξ with $d(\xi) = \bar{d}$, the baseline cost-to-load is greater than the proposed cost-to-load because of the strict convexity of the cost function of the conventional generators, while the reverse is true in an excess scenario with $d(\xi) = \bar{d}$.

For each scenario ξ, exploiting (4), (17), and (18), we have:

$$\begin{aligned} &\mathbb{E}_\Xi[\text{CTL}^{\text{baseline}}(\xi)] \\ &= \mathbb{E}_\Xi\big[\alpha x_g^{DAM^*} + c_g'((d-\lambda^{TOT})(\xi))(d - x_g^{DAM^*})\big] \\ &= \mathbb{E}_\Xi[c_g'((d-\lambda^{TOT})(\xi))\, d] \\ &= \mathbb{E}_\Xi[\text{CTL}^{\text{proposed}}(\xi)]. \end{aligned} \qquad (19)$$

The following proposition is immediate.

Proposition 5.3. *Assume Assumptions 1–7 hold. The expected cost-to-load is the same in the proposed market and the baseline market.*

6. Profit-Optimal Linear Cost Function for Renewables

While the equivalence results in Section 5 establish that the proposed market design preserves system-wide efficiency metrics relative to the baseline model, it remains to be examined how renewable bidders may strategically adjust their bids to optimize individual profitability. We assume the renewables use a linear cost function of the form $\hat{\alpha} x^{DAM}_{r_{TOT}}$ + const. We show that if the renewable generators wish to maximize their aggregate expected profit, they should choose $\hat{\alpha} > \alpha$ if the conventional generators' cost function $c_g(x)$ is strictly convex and monotonically increasing.

With a linear renewable cost function, it follows from the solution (9) to (8) that the optimal choice is to take x^{DAM}_g to satisfy $c'_g(x^{DAM}_g) = \hat{\alpha}$. This then gives $x^{DAM}_{r_{TOT}} = \bar{d} - x^{DAM}_g$ and the DAM revenue for the renewable generators is $\hat{\alpha}(\bar{d} - x^{DAM}_g) = \hat{\alpha} x^{DAM}_{r_{TOT}}$. The expected profit for the renewable generators in the RTM is expressed as follows

$$\begin{aligned} &\text{Expected RTM profit for renewables} \\ &\quad = \mathbb{E}_{\Xi}[c'_g((d-\lambda^{TOT})(\xi))(\lambda^{TOT}(\xi) - x^{DAM}_{r_{TOT}})] \\ &\quad = \mathbb{E}_{\Xi}[c'_g((d-\lambda^{TOT})(\xi))\lambda^{TOT}(\xi)] \\ &\qquad\quad - \mathbb{E}_{\Xi}[c'_g((d-\lambda^{TOT})(\xi))]x^{DAM}_{r_{TOT}} \\ &\quad = \textstyle\sum_{r\in R} \beta_r - \alpha x^{DAM}_{r_{TOT}} \ \text{ from (4)-(5).} \end{aligned} \tag{20}$$

Therefore, the overall expected profit is

$$\begin{aligned} &\text{Expected DAM+RTM profit for renewables} \\ &\quad = \ \textstyle\sum_{r\in R} \beta_r + (\hat{\alpha} - \alpha) x^{DAM}_{r_{TOT}}. \end{aligned} \tag{21}$$

Note that $x^{DAM}_{r_{TOT}}$ decreases as $\hat{\alpha}$ increases, and $x^{DAM}_{r_{TOT}} = 0$ for $\hat{\alpha} \geq c'_g(d)$. Hence the renewables' profit is maximized for some $\hat{\alpha}$ satisfying $\alpha \leq \hat{\alpha} \leq c'_g(\bar{d})$, so $x^{DAM}_{r_{TOT}}(\hat{\alpha}) \leq x^{DAM}_{r_{TOT}}(\alpha)$. If $c_g(x_g)$ is strictly convex then these inequalities are strict. Setting the derivative with respect to $\hat{\alpha}$ of (21) equal to zero shows that $\hat{\alpha}$ and $x^{DAM}_{r_{TOT}}(\hat{\alpha})$ must satisfy

$$0 = x^{DAM}_{r_{TOT}}(\hat{\alpha}) + (\hat{\alpha} - \alpha)\frac{d}{d\hat{\alpha}} x^{DAM}_{r_{TOT}}(\hat{\alpha}). \tag{22}$$

We have $c'_g(x^{DAM}_g) = \hat{\alpha}$ from (9), so under Assumption 7, we have $x^{DAM}_g = (\hat{\alpha} - b)/(2a)$, so $x^{DAM}_{r_{TOT}} = \bar{d} - (\hat{\alpha} - b)/(2a)$. Substituting in (22) and rearranging gives the optimal choice for $\hat{\alpha}$.

Proposition 6.1. *Assume Assumptions 1–7 hold. The choice of α that maximizes the profit of the renewables is*

$$\hat{\alpha} = a\bar{d} + \frac{1}{2}(b + \alpha). \tag{23}$$

This choice of $\hat{\alpha}$ ensures x^{DAM}_g is in the range $(x^{DAM^*}_g, \bar{d})$, since $\hat{\alpha} = \frac{1}{2}(\alpha + c'_g(\bar{d}))$.

It is apparent that the expected cost-to-load associated to $\hat{\alpha}$ is greater than that of α, since $\hat{\alpha} > \alpha$. This exposes the natural trade-off that maximizing renewable revenue would lead to an increase in the cost-to-load.

7. Cost and Profit Variability Analysis

The proposed market modifies renewable participation under day-ahead and real-time settlement. To understand the impact of electricity market redesign on suppliers and consumers, we analyze variance of profit for both conventional and renewable generators and cost-to-load under both the baseline and proposed markets. The variance difference between the baseline and proposed market reveals whether the new market design increases or decreases risk for the corresponding party (generator or consumer), which is analyzed in the following subsections.

7.1. Conventional Generator Profit

The DAM and RTM values of the optimal x_g are identical, and the payments to the conventional generators remain the same in both market models. As a result, the profits of the conventional generators do not differ between the proposed and baseline markets. Hence, the variance of profits for conventional generators is the same in both markets.

7.2. Renewable Generator Profit

Taking the difference of the variances of (14) and (15), and omitting the scenario index ξ^k for notational simplicity, we obtain

$$\begin{aligned} &\mathrm{Var}(\mathrm{profit}^{\mathrm{baseline}}_{r_{TOT}}) - \mathrm{Var}(\mathrm{profit}^{\mathrm{proposed}}_{r_{TOT}}) \\ &\quad = 2\, x^{DAM}_{r_{TOT}} \mathbb{E}_{\Xi}[c'_g(d - \lambda^{TOT})^2 \lambda^{TOT}] \\ &\qquad - 2\, x^{DAM}_{r_{TOT}} \alpha\, \mathbb{E}_{\Xi}[c'_g(d - \lambda^{TOT})\lambda^{TOT}] \\ &\qquad - (x^{DAM}_{r_{TOT}})^2\, \mathrm{Var}(c'_g(d - \lambda^{TOT})). \end{aligned}$$

Let

$$p = c'_g(d - \lambda^{TOT}), \ x^{DAM}_{r_{TOT}} = \bar{\lambda} = \mathbb{E}_{\Xi}(\lambda^{TOT}),$$

where $\bar{\lambda} = \mathbb{E}_{\Xi}[\lambda^{TOT}]$ is defined in section 5. We have the following result (proof in Appendix D).

Proposition 7.1. *The difference in the variance of renewable generator profit between the baseline and the proposed markets can be expressed as*

$$\begin{aligned} &\mathrm{Var}(\mathrm{profit}^{\mathrm{baseline}}_{r_{TOT}}) - \mathrm{Var}(\mathrm{profit}^{\mathrm{proposed}}_{r_{TOT}}) \\ &\quad = \bar{\lambda}^2\, \mathrm{Var}(p) + 2\bar{\lambda}\alpha\, \mathrm{Cov}(\lambda^{TOT}, p) \\ &\qquad + 2\bar{\lambda}\, \mathrm{Cov}(\lambda^{TOT}, (p - \alpha)^2). \end{aligned} \tag{24}$$

The individual terms contributing to the variance difference of (24) and the corresponding total variance difference between the baseline and proposed markets, calculated using data from Electricity Reliability Council of Texas (2024a,b,c), are summarized in Table 5. Since $\mathrm{Var}(p) \geq 0$, the first term in (24) is non-negative. After analyzing the ERCOT 2024 data, real-time renewable output (λ^{TOT}) and RTM price (p) are observed to be negatively correlated, implying that higher renewable generation is typically associated with lower RTM prices, as intuitively expected. This implies that the second term in (24), $2\bar{\lambda}\alpha\ \mathrm{Cov}(\lambda^{TOT}, p)$, is negative. The third term, $2\bar{\lambda}\,\mathrm{Cov}(\lambda^{TOT}, (p-\alpha)^2)$ is observed to yield a negative value in the ERCOT 2024 data. This implies that larger price fluctuations tend to correspond to smaller values of λ^{TOT}.

Overall, the difference in variances between the baseline and proposed markets for renewable profit tend to be both positive and negative values across different hours. From the ERCOT data, we observe that during peak-demand hours, the proposed market exhibits lower variance compared to the baseline market, suggesting that the proposed system is more robust for renewable profit during these high-demand periods. On the other hand, some off-peak periods show negative variance differences, indicating that the proposed framework might lead to increased profit variability for renewables under lightly loaded market conditions.

7.3. Cost-to-Load

Omitting the scenario index ξ^k from (17) and (18), cost-to-load for the baseline market is obtained as,

$$\mathrm{CTL}^{\mathrm{baseline}} = \mathrm{CTL}^{\mathrm{proposed}} + \bar{\lambda}(c_g'(d - \lambda^{TOT}) - \alpha). \tag{25}$$

We have the following result (proof in Appendix E).

Proposition 7.2. *The difference in the variance of cost-to-load between the baseline market and the proposed market can be expressed as*

$$\begin{aligned} &\mathrm{Var}(\mathrm{CTL}^{\mathrm{baseline}}) - \mathrm{Var}(\mathrm{CTL}^{\mathrm{proposed}}) \\ &= \bar{\lambda}^2\,\mathrm{Var}(p) + 2\bar{\lambda}\alpha\,\mathrm{Cov}(d,p) + 2\bar{\lambda}\,\mathrm{Cov}(d,(p-\alpha)^2). \end{aligned} \tag{26}$$

Table 6 summarizes the total variance difference of cost-to-load between the baseline and proposed markets, as well as the individual terms that contribute to the variance difference in (26). The data used in this analysis is again derived from Electricity Reliability Council of Texas (2024a,b,c,d). Since $\mathrm{Var}(p) \geq 0$, the first term in (26) is non-negative. The analysis of ERCOT 2024 data shows that real-time demand (d) and RTM price (p) are positively correlated, as intuitively expected. This implies that the second term in (26), $2\bar{\lambda}\alpha\,\mathrm{Cov}(d,p)$ is positive. The third term, $2\bar{\lambda}\,\mathrm{Cov}(d,(p-\alpha)^2)$ is found to be positive for most hours using ERCOT 2024 dataset indicating that higher demand is associated with greater price variability. Consequently, the variance difference for cost-to-load between the baseline and the proposed market is always positive, as seen in the last column of Table 6. This implies that as compared to the baseline, the proposed market generally presents a lower price risk for consumers, where the price risk originates from uncertainty in the demand and the renewable generation. When the demand is well predicted (i.e., the price risk arises solely from the uncertainty of renewable generation), this variance difference is strictly positive as established below.

An equivalent representation of (26), derived in Appendix E, is

$$\begin{aligned} &\mathrm{Var}(\mathrm{CTL}^{\mathrm{baseline}}) - \mathrm{Var}(\mathrm{CTL}^{\mathrm{proposed}}) \\ &\quad = \bar{\lambda}^2\ \mathrm{Var}(p) + 2\bar{\lambda}\ \mathrm{Cov}((d-\bar{d})p,\ p). \end{aligned} \tag{27}$$

Empirical data from Electric Reliability Council of Texas (ERCOT) (2024) indicate that the deviation $(d - \bar{d})$ remains small, typically within 3.5%. Consequently, the term $2\bar{\lambda}\,\mathrm{Cov}((d-\bar{d})p,\ p)$ is empirically small relative to $\bar{\lambda}^2\,\mathrm{Var}(p)$ leading to the following corollary.

Corollary 7.3. *If $(d - \bar{d})$ remains sufficiently small, the non-negative term $\bar{\lambda}^2\,\mathrm{Var}(p)$ dominates in (27), which implies*

$$\mathrm{Var}(\mathrm{CTL}^{\mathrm{baseline}}) - \mathrm{Var}(\mathrm{CTL}^{\mathrm{proposed}}) > 0.$$

Therefore, the proposed design yields a lower variance in cost-to-load than the baseline market. If demand is perfectly predicted, i.e., $d = \bar{d}$, then from (17),

$$\mathrm{CTL}^{\mathrm{proposed}} = \alpha\bar{d},$$

which is constant. Hence, $\mathrm{Var}(\mathrm{CTL}^{\mathrm{proposed}}) = 0$.

8. Risk-adjusted Cost Curves

While Section 6 focused on revenue-maximizing bidding strategies for renewable generators, it is equally important to account for risk exposure arising from real-time shortfalls. To this end, we next develop risk-adjusted cost curves to reduce real-time vulnerability. The study of risk-averse and coherent risk measures is an active area of research with an early paper being Artzner, Delbaen, Eber and Heath (1999). Conditional Value-at-Risk (CVaR) (Rockafellar and Uryasev, 2000) is a coherent risk measure through which a renewable could incorporate its risk of a large RTM charge into its cost curve. In such an approach, a renewable is concerned with the expectation of the worst ν proportion of scenarios that can result in a shortfall. For ease of exposition, we assume the scenarios are equally likely; the framework however extends to scenarios being drawn from an arbitrary distribution. We divide our analysis into two cases: (a) when the CVaR metric captures the risk of shortfall of each renewable individually (*disaggregated* case), (b) when CVaR captures the risk of shortfall in the total renewable supply (*aggregated* case). In case (a), an individual renewable may have a piecewise linear CVaR cost function, while in case (b), the aggregated renewables possess a linear CVaR cost function.

8.1. Disaggregated CVaR

For a given $x_{r,CVaR}^{DAM}$, the contribution of a particular scenario ξ^k to the CVaR of renewable r can be written as

$$\begin{aligned} &\check{V}_r(x_{r,CVaR}^{DAM}, \xi^k) = \\ &\begin{cases} \check{c}_r(x_{r,CVaR}^{DAM}, \xi^k) & \text{if } \check{c}_r(x_{r,CVaR}^{DAM}, \xi^k) \geq z_r^{\nu}\left(x_{r,CVaR}^{DAM}\right), \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \tag{28}$$

where $z_r^{\nu}(x_{r,CVaR}^{DAM})$ is the cutoff for a scenario to be one of the ν proportion of scenarios with the highest cost (informally, the "bad scenarios"), and $\check{c}_r(x_{r,CVaR}^{DAM}, \xi^k)$ is defined in (3). The function $\check{V}_r\left(x_{r,CVaR}^{DAM}, \xi^k\right)$ is linear as long as $x_{r,CVaR}^{DAM}$ is in the range where ξ^k is a bad scenario, and zero otherwise. The overall CVaR for renewable r is then

$$\check{V}_r(x_{r,CVaR}^{DAM}) = \frac{1}{\nu}\sum_k \check{V}_r(x_{r,CVaR}^{DAM}, \xi^k). \tag{29}$$

In general, CVaR is a convex and monotonically increasing function. If the number of scenarios is finite, it is piecewise linear. We give an example to show the calculation of CVaR.

Example We consider a renewable, with 4 scenarios (k) as follows. For this example, as $\lambda_r(\xi_r^k)$ increases, the total renewable production λ^{TOT} decreases. The two illustrative points (0 and 10) correspond to the minimum and maximum renewable production levels, respectively.

k	$\lambda_r(\xi_r^k)$	$c_g'(d-\lambda^{TOT})$	$\check{c}_r(0,\xi_r^k)$	$\check{c}_r(10,\xi_r^k)$
1	0	2	0	20
2	5	10	-50	50
3	5	20	-100	100
4	10	20	-200	0

Assume each scenario is equally likely, and that CVaR is calculated using the two worst scenarios. The CVaR $\check{V}_r(x_{r,CVaR}^{DAM})$ is then as listed in the following table and illustrated in Figure 2.

$x_{r,CVaR}^{DAM}$ range	worst 2 scenarios	CVaR
$[0,5]$	$\{1,2\}$	$6x_{r,CVaR}^{DAM}-25$
$[5,6.25]$	$\{1,3\}$	$11x_{r,CVaR}^{DAM}-50$
$[6.25,10]$	$\{2,3\}$	$15x_{r,CVaR}^{DAM}-75$

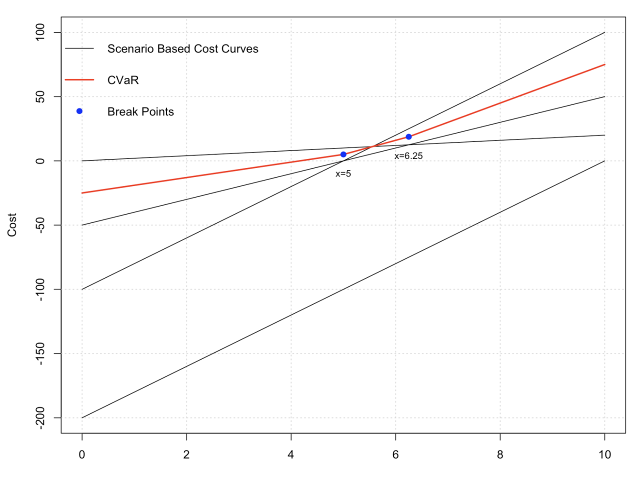


Figure 2: Plot of cost for four scenarios (black lines). CVaR $\check{V}_r(x)$ plotted in red.

Hence $\check{V}_r(x_{r,CVaR}^{DAM})$ is a piecewise linear, monotonically increasing, and convex function for this example.

Note that for this example, we have $\alpha = 13$, so the slope of $\check{V}_r(x_{r,CVaR}^{DAM})$ is below α for $x_{r,CVaR}^{DAM} \leq 5$ and above for $x_{r,CVaR}^{DAM} > 5$. It is possible to construct examples where the slope of $\check{V}_r(x_{r,CVaR}^{DAM})$ is below α for all $x_{r,CVaR}^{DAM}$, and others where the slope of $\check{V}_r(x_{r,CVaR}^{DAM})$ is above α for all $x_{r,CVaR}^{DAM}$. $\square$

Following Assumption 6, we set the CVaR cost function submitted to the ISO as

$$V_r(x_{r,CVaR}^{DAM}) := \check{V}_r(x_{r,CVaR}^{DAM}) + k_r, \tag{30}$$

where k_r is a constant chosen so that Assumption 6 holds. (In the example, $k_r = 25$.) We now give a sufficient condition for $V_r(x_{r,CVaR}^{DAM})$ to be a linear function.

Proposition 8.1. *For a given renewable r, and for each pair of scenarios ξ^i and ξ^j, assume that if $\lambda_r(\xi^i) \geq \lambda_r(\xi^j)$ then $(d-\lambda^{TOT})(\xi^i) \leq (d-\lambda^{TOT})(\xi^j)$, so the price decreases as renewable production increases. It then follows that $V_r(x_{r,CVaR}^{DAM})$ is a linear function.*

The proof of this proposition follows from the following elementary lemma, which shows that the set of bad scenarios is the same for all $x_{r,CVaR}^{DAM} \geq 0$ in the setting of the proposition.

Lemma 8.2. *Consider two linear functions of x, namely $m_1x-\lambda_1$ and $m_2x-\lambda_2$. Assume $\lambda_1 \geq \lambda_2$ and $m_1 \leq m_2$. Then $m_1x-\lambda_1-[m_2x-\lambda_2] \leq 0$ for all $x \geq 0$.*

Lemma 8.2 says that the lines do not cross for $x \geq 0$. In the context of the proposition, m_1 and m_2 represent the prices $c_g'(d-\lambda^{TOT})$ for the two scenarios. It is worth noting that the renewable cost curve shown in Figure 2 does not satisfy the assumptions of Proposition 8.1, and therefore it is only piecewise linear.

In practice, a cost function is often a convex combination of the expected cost and the CVaR. Let the CVaR be weighted by η. For a given scenario ξ^k and a given renewable generator r with accepted DAM bid of $x_{r,CVaR}^{DAM}$, exploiting Assumptions 4 and 5, the expected risk-adjusted RTM cost for the renewable r is

$$\begin{aligned} c_{r,\eta}&(x_{r,CVaR}^{DAM}) \\ &= (1-\eta)c_r(x_{r,CVaR}^{DAM}) + \eta V_r(x_{r,CVaR}^{DAM}) \\ &= (1-\eta)\alpha x_{r,CVaR}^{DAM} + \eta V_r(x_{r,CVaR}^{DAM}), \end{aligned} \tag{31}$$

where the last step exploits equation 7. In general, this is a monotonically increasing convex function of $x_{r,CVaR}^{DAM}$ with $c_{r,\eta}(0) = 0$. If there is a finite number of scenarios then it is piecewise linear.

8.2. Aggregated CVaR

We now consider the case where the renewable generators, both solar and wind energy suppliers with different forecasted scenarios, act as a coalition and submit a single cost curve. In practice, this situation is realized when the renewables with excess generation will cover for others with shortfall outside of the market, and therefore, the CVaR function only needs to capture the risk of the aggregate shortfall across all renewables. In this case, the CVaR for scenario ξ^k is

$$\check{V}_{rTOT}(x_{rTOT,CVaR}^{DAM},\xi^k) = \begin{cases} \frac{1}{v}\check{c}_{rTOT}(x_{rTOT,CVaR}^{DAM},\xi^k) \\ \quad \text{if } \check{c}_{rTOT}(x_{rTOT,CVaR}^{DAM},\xi^k) \geq z_{rTOT}^{v}\left(x_{rTOT,CVaR}^{DAM}\right) \\ 0 \text{ otherwise.} \end{cases} \tag{32}$$

Note that the slope $c_g'\big((d-\lambda^{TOT})(\xi^k)\big)$ is monotonically decreasing as $(d-\lambda^{TOT})(\xi^k)$ decreases. Thus, from

Lemma 8.2, the set of bad scenarios is the same for all nonnegative $x^{DAM}_{r^{TOT},CVaR}$; in particular, the bad scenarios are those with the largest shortfall $(d-\lambda^{TOT})(\xi^k)$. Consequently, $V_{r^{TOT}}(x^{DAM}_{r^{TOT},CVaR})$ is a linear function of $x^{DAM}_{r^{TOT},CVaR}$, which we write as

$$V_{r^{TOT}}(x^{DAM}_{r^{TOT},CVaR}) = \alpha^{CVaR} x^{DAM}_{r^{TOT},CVaR} - \beta^{CVaR} \quad (33)$$

with

$$\begin{aligned} &c_{r^{TOT},\eta}(x^{DAM}_{r^{TOT},CVaR}) \\ &\quad = \big((1-\eta)\alpha + \eta\alpha^{CVaR}\big)x^{DAM}_{r^{TOT},CVaR} \\ &\qquad - \big((1-\eta)\beta_{r^{TOT}} + \eta\beta^{CVaR}\big). \end{aligned} \quad (34)$$

Since the CVaR is calculated using the bad scenarios which have the largest slopes, we obtain $\alpha^{CVaR} \geq \alpha$.

From the linearity of the renewable cost function, the ISO solves a problem similar to (8). The optimal solution $x^{DAM*}_{g,CVaR}$ will have the same structure as (9), with α replaced by $(1-\eta)\alpha + \eta\alpha^{CVaR}$. Since $\alpha^{CVaR} \geq \alpha$, we have $x^{DAM}_{g,CVaR} \geq x^{DAM}_{g}$ and $x^{DAM}_{r^{TOT},CVaR} \leq x^{DAM}_{r^{TOT}}$.

The results of this section are summarized in the following proposition.

Proposition 8.3. *Assume Assumptions 1–6 hold. Assume the renewables submit a single CVaR bid as a coalition. The CVaR bid function is the linear function given in (33). The slope of this linear function is greater than α, and $x^{DAM}_{g,CVaR} \geq x^{DAM}_{g}$, so the cost-to-load is larger than in the proposed model (8).*

9. Evaluation

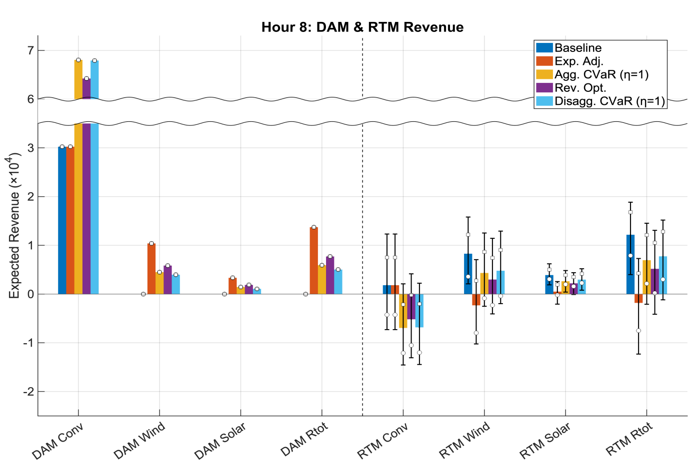
We used MATLAB to simulate market outcomes using the 2024 Hourly Aggregated Wind and Solar Output dataset from the Electric Reliability Council of Texas (ERCOT) (Electricity Reliability Council of Texas, 2024c). We merged the wind and solar data, isolating weekday data for July. For each hour h ($h = 0, \dots, 23$), we selected the first 10 even-indexed weekdays as the calibration dataset to calculate the α and β_r parameters. We then evaluated our approach using the first 10 odd-indexed weekdays. All data was proportionally scaled based on the average hourly load of the even-indexed weekdays. In effect, we assume d_h is accurately forecast, which allows us to study the impact of the variability in renewable generation which is the key focus of the paper.

9.1. Plotting Metrics of Different Models

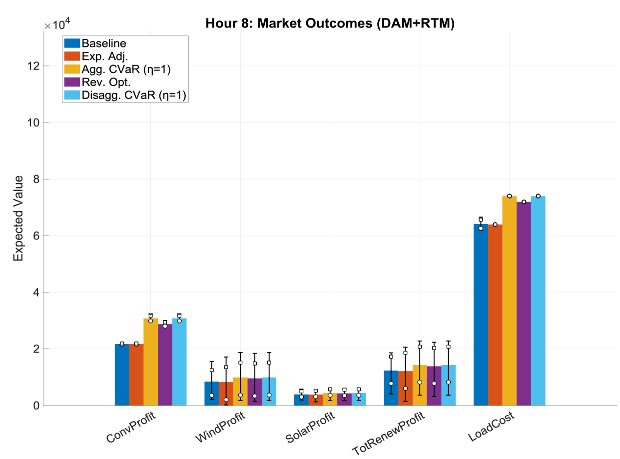
In the baseline market, the DAM price is the conventional marginal cost evaluated at expected net load, whereas in the proposed market the DAM price parameter α is the expected marginal cost from the calibration sample; RTM settlements in both cases are determined by realized marginal costs.

Figure 9.1 compares market outcomes under the baseline, expectation-adjusted, revenue-optimal, and CVaR-based strategies. Across all hours, conventional profit is unchanged across market designs, consistent with the theory. We therefore focus on two representative hours, Hour 8 and Hour 20, to show how renewable profits and cost-to-load differ between shortfall and excess scenarios.

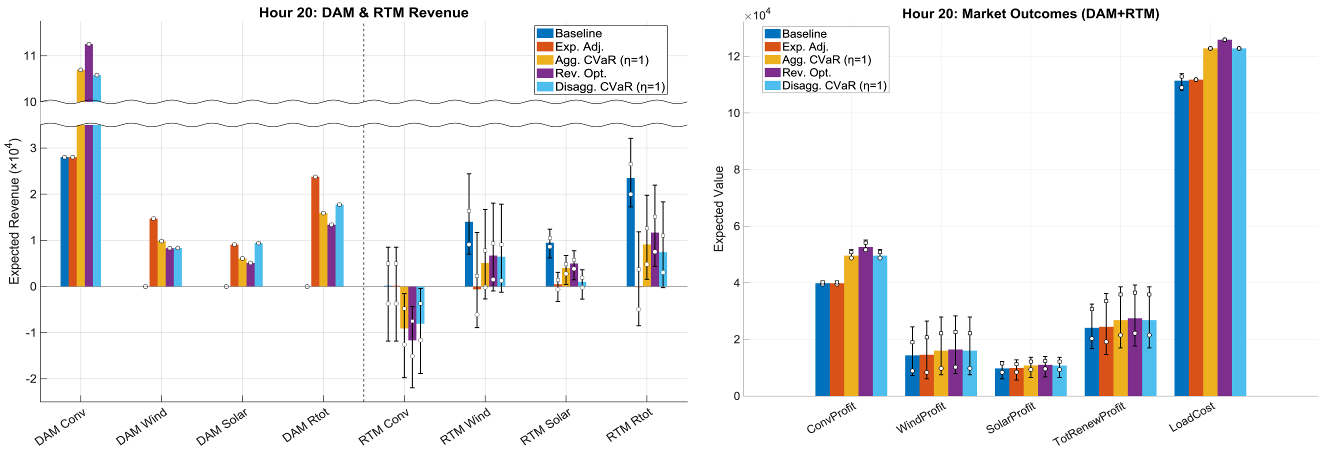
Hour 8 [Figure 9.1(b)] is a total shortfall case. Because realized renewable output falls below DAM commitments, renewables must buy energy in real time, which lowers renewable profit in the proposed market. Consistent with the theory, the corresponding cost-to-load is lower than in the baseline market.



(a)



(b)



(c) (d)

Figure 3: Comparison of expected DAM and RTM revenues across representative morning (Hour 8) and evening (Hour 20) intervals. Bars represent expected values, with error bars indicating the full range (min–max) and markers denoting the 25th/75th percentiles. Strategies include: **Baseline** (status-quo), **Exp-Adj.** (expectation-adjusted), **Rev. Opt.** (revenue-optimal), and **Agg./Disagg. CVaR** ($\eta = 1$) (aggregated and disaggregated conditional value-at-risk with high risk aversion). Note that the vertical axis includes a scale break (indicated by wavy lines) to improve readability.

Hour 20 [Figure 9.1(d)] is a total excess case. In this setting, renewables earn more in the proposed market by selling excess energy in real time, and the cost-to-load is correspondingly higher than in the baseline market.

Figure 9.1(a)–(d) also shows a clear risk–return trade-off. The expectation-adjusted strategy yields the highest DAM revenue but performs poorly in RTM during shortfall hours, whereas CVaR-based strategies reduce DAM commitments and provide greater downside protection.

Aggregation further changes the composition of renewable commitments. In Hour 8, aggregation supports higher total DAM commitments through diversification, while in Hour 20 it shifts commitment volume from solar to wind to hedge evening volatility.

Table 1 shows that CVaR-based adjustments are most conservative in the morning and evening hours, whereas the revenue-optimal adjustment is largest around midday. All adjusted values remain below the marginal-cost benchmark, reflecting the trade-off between risk control and higher expected revenue.

Table 1
Hourly values of α under different cost-curve adjustment schemes (rounded to two decimals).

Hour	α_{orig}	$\alpha_{\text{CVaR},\eta=0.5}$	$\alpha_{\text{CVaR},\eta=1}$	$\hat{\alpha}$	$c'_g(d)$
0	1.38	1.49	**1.60**	1.59	1.80
1	1.29	1.41	**1.52**	1.50	1.71
2	1.24	1.36	**1.47**	1.44	1.64
3	1.21	1.32	**1.43**	1.39	1.58
4	1.19	1.30	**1.41**	1.37	1.54
5	1.21	1.31	**1.41**	1.37	1.53
6	1.25	1.34	**1.44**	1.39	1.54
7	1.29	1.38	**1.47**	1.43	1.56
8	1.27	1.37	**1.47**	1.43	1.58
9	1.20	1.32	**1.43**	**1.43**	1.66
10	1.19	1.31	1.43	**1.47**	1.74
11	1.24	1.34	1.44	**1.53**	1.83
12	1.31	1.39	1.47	**1.61**	1.91
13	1.38	1.45	1.51	**1.69**	1.99
14	1.43	1.49	1.55	**1.75**	2.06
15	1.47	1.55	1.62	**1.79**	2.11
16	1.52	1.59	1.66	**1.83**	2.15
17	1.55	1.62	1.69	**1.86**	2.17
18	1.56	1.64	1.72	**1.86**	2.17
19	1.58	1.67	1.76	**1.86**	2.14
20	1.66	1.74	1.82	**1.87**	2.08
21	1.72	1.79	**1.86**	**1.86**	2.01
22	1.64	1.72	**1.80**	**1.80**	1.96
23	1.51	1.62	**1.73**	1.69	1.87

9.2. An Illustration of Calculating Cost Curves

Here, we assume a realistic conventional cost function

$$c_g(x) = 0.0000146256\, x^2 + 0.107617\, x$$

derived from analyzing actual conventional generation cost data, with the fixed cost removed to satisfy Assumption 6. Then

$$c'_g(x) = 0.0000292512\, x + 0.107617.$$

Consider the following sample data in Table 2 of Hour 8 and Hour 20. In this example, DAM commitments are set equal to expected renewable generation, matching the baseline-market assumption. The resulting profits and cost-to-load are summarized in Table 3. Hour 20 is a total excess case ($\bar{\lambda} < \lambda$), so renewable profits and cost-to-load are both higher in the proposed market, while conventional profit is unchanged. Hour 8 is a total shortfall case, and the direction of these differences is reversed.

10. Concluding Remarks

In this paper, we have analyzed an electricity market where renewable generators are allowed to bid cost curves

Table 2
Example Data with Two Renewable Generators (one Solar and one Wind).

Parameter	Hour 8 (MW)	Hour 20 (MW)
d (Load)	50468	67367
w (Wind DAM Commitment)	8197	8861
s (Solar DAM Commitment)	2629	5460
λ_w (Wind Gen. in RTM)	7091	9178
λ_s (Solar Gen. in RTM)	3131	5995
$\bar{\lambda}$ (Agg. Ren. DAM Commitment)	10826	14321
λ (Agg. Ren. Gen.)	10222	15173

Table 3
Comparison of Profits and Cost-to-Load (CTL) in the Baseline and Proposed Markets.

Metric	**Baseline($)**		**Proposed($)**	
	Hour 8	Hour 20	Hour 8	Hour 20
Conv. Profit	21721	39847	21721	39847
Wind Profit	8425	14355	8280	14576
Solar Profit	3911	9744	3865	9880
Agg. Ren. Profit	12336	24099	12145	24456
CTL	64255	111462	64094	111829

into the DAM, just like conventional generators. We analytically derive the expressions for the renewable cost curves so that they take into account the buying (in case of shortfall) and selling (in case of excess) of electricity in the RTM, as needed by the renewable to meet its DAM commitment and utilize its full supply without curtailment. We theoretically establish that compared to current practice, the proposed market preserves overall conventional and renewable generator profits, and cost-to-load performance metrics, on the average. Conventional-generator profits coincide between the proposed and baseline market designs in every uncertainty scenario, but the variability in renewable generator profit may be higher or lower in the proposed market depending on operating conditions. However, the proposed market results in lower variability in the cost-to-load, under certain reasonable conditions that are validated using real market data. When the demand is well predicted, then total cost-to-load in the proposed market is deterministic, i.e., any impact of the variability in the renewable supply is not passed on to the consumers. Scenario-level outcomes nonetheless redistribute the generator revenues between shortfall and excess realizations. We validate these theoretical results using a data set from the Texas grid, considering scenarios where the renewable forecast may not be perfect.

Other than protecting loads (consumers) from price fluctuations due to the variability in renewable supply, there are several practical benefits of the proposed market with renewable bidders. First, it pushes some of the complexity of market operations under renewable penetration to the edge (namely, the renewables), allowing the ISO to clear the market by solving a deterministic UC problem, while the stochasticity is captured in the calculation of the renewable cost curves that are determined by the renewable generators individually. This avoids the need for the ISO to solve the computationally expensive stochastic UC problem (Zheng et al., 2015; Håberg, 2019). Second, by allowing renewables to bid cost curves instead of a single generation amount, it allows them more flexibility to incorporate additional risk-tolerance measures into the cost curves. This allows renewables to better manage their risk in the two-stage market participation than currently possible. Incorporating Conditional Value-at-Risk (CVaR) into renewable bidding provides a structured means to manage downside risk. By steepening bid curves, CVaR reduces DAM commitments and mitigates RTM penalties in adverse scenarios. The results reveal a clear trade-off: greater risk aversion enhances revenue stability but increases load costs. Aggregated CVaR smooths portfolio outcomes by capturing cross-unit correlations, while disaggregated CVaR retains individual sensitivities and yields higher RTM variability, highlighting how CVaR systematically modulates commitment behavior under uncertainty.

There are several research directions that merit further investigation. Our equivalence results depend critically on the quadratic nature of the conventional cost curves. While this is reasonable and widely assumed in practice, extension to more general convex cost curves remains open. Further, note that we assume total flexibility (large ramp rates) of the conventional generators (Assumption 4). The commitments of some types of conventional generators cannot be changed easily in real-time, or they may have ramp-up/down constraints. Such a scenario would result in renewable curtailment, and needs to be investigated further. For future work, we also intend to look at multi-period models and incorporate ramping, storage, startup, and shutdown constraints.

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Table 4
Selected Texas grid Market Data (unit in MW).

Scenarios	Wind	Solar	Agg-Renew.
ξ^1	10443	4319	14762
ξ^2	10967	4149	15116
ξ^3	12876	3921	16797
ξ^4	2239	4141	6380
ξ^5	6910	4100	11010
ξ^6	14426	2785	17211
ξ^7	6195	2647	8842
ξ^8	2993	2471	5464
ξ^9	1418	1273	2691
ξ^{10}	2445	1501	3946

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A. Sample Calculation of α and β_r

Consider the data of Hour 8 in Table 4 with 10 scenarios with $d = 50468$. If we use a realistic conventional cost function

$$c_g(x) = 0.0000146256\, x^2 + 0.107617\, x + 1666.72,$$

with $c'_g(x) = 0.0000292512\, x + 0.107617$, we have

$$\begin{aligned}
\alpha &= \mathbb{E}_\Xi\left[c'_g((d-\lambda^{TOT})(\xi))\right] \\
&= \tfrac{1}{10}\textstyle\sum_{k=1}^{10} c'_g\big((d-\lambda^{TOT})(\xi^k)\big) \\
&= \tfrac{1}{10}\big(0.0000292512 \\
&\quad \times(50468-14762)+0.107617 \\
&\quad + \dots \\
&\quad +0.0000292512 \\
&\quad \times(50468-3946)+0.107617\big) \\
&= \tfrac{1}{10}(1.15+\dots+1.47) \\
&\approx 1.28,
\end{aligned} \tag{35}$$

$$\begin{aligned}\beta_w &= \mathbb{E}_\Xi\left[c_g'((d-\lambda^{TOT})(\xi))\,\lambda_w(\xi)\right]\\ &= \tfrac{1}{10}\textstyle\sum_{k=1}^{10} c_g'((d-\lambda^{TOT})(\xi^k))\lambda_w(\xi^k)\\ &= \tfrac{1}{10}(1.15\times 10443 + \ldots\\ &\quad + 1.47\times 2445) \approx 8419,\end{aligned} \tag{36}$$

$$\begin{aligned}\beta_s &= \mathbb{E}_\Xi\left[c_g'((d-\lambda^{TOT})(\xi))\,\lambda_s(\xi)\right]\\ &= \tfrac{1}{10}\textstyle\sum_{k=1}^{10} c_g'((d-\lambda^{TOT})(\xi^k))\lambda_s(\xi^k)\\ &= \tfrac{1}{10}(1.15\times 4319 + \ldots\\ &\quad +1.47\times 1501) \approx 3910,\end{aligned} \tag{37}$$

assuming all 10 scenarios are equally likely.

B. Proof of Lemma 4.2

Proof: From (12), the conventional generators' profit would be

$$\begin{aligned}\text{profit}_g(\xi^k) &= \alpha x_g^{DAM*} - c_g((d-\lambda^{TOT})(\xi^k))\\ &\quad + c_g'((d-\lambda^{TOT})(\xi^k))\\ &\quad \times((d-\lambda^{TOT})(\xi^k) - x_g^{DAM*})\\ &= \alpha x_g^{DAM*} - c_g(x_g^{DAM*})\\ &\quad + c_g(x_g^{DAM*}) - c_g((d-\lambda^{TOT})(\xi^k))\\ &\quad + c_g'((d-\lambda^{TOT})(\xi^k))\\ &\quad \times((d-\lambda^{TOT})(\xi^k) - x_g^{DAM*})\\ &\geq \alpha x_g^{DAM*} - c_g(x_g^{DAM*})\\ &= \text{profit}_g^{DAM} \text{ from (13).}\end{aligned} \tag{38}$$

The inequality follows from the convexity of $c_g(x_g)$ of Assumption 2, which gives

$$\begin{aligned}c_g(x_g^{DAM*}) &\geq c_g((d-\lambda^{TOT})(\xi^k))\\ &\quad + c_g'((d-\lambda^{TOT})(\xi^k))\left[x_g^{DAM*} - ((d-\lambda^{TOT})(\xi^k))\right],\end{aligned} \tag{39}$$

showing the required result. □

C. Proof of Proposition 5.1

Proof: In the baseline market practice, we have $x_g^{DAM*}(\text{baseline}) = \mathbb{E}_\Xi(d-\lambda^{TOT})$, whereas in our proposed market practice, $x_g^{DAM*}(\text{proposed})$ satisfies $c_g'(x_g^{DAM*}(\text{proposed})) = \alpha = \mathbb{E}_\Xi(c_g'(d-\lambda^{TOT}))$ (from (4) and (9)). Assuming that $c_g(x_g) = ax_g^2 + bx_g + c$ with a, b, and c being some constants, we have $c_g'(x_g) = 2ax_g + b$. Note that the cost function $c_g(x_g)$ for the conventional generators is the same in both the baseline and proposed market. Then,

$$\begin{aligned}&\mathbb{E}_\Xi(c_g'(d-\lambda^{TOT}))\\ &\quad = \mathbb{E}_\Xi(2a(d-\lambda^{TOT}) + b)\\ &\quad = 2a(\mathbb{E}_\Xi(d-\lambda^{TOT})) + b\\ &\quad = 2a\left(x_g^{DAM*}(\text{baseline})\right) + b.\end{aligned} \tag{40}$$

We also have

$$\begin{aligned}&\mathbb{E}_\Xi(c_g'(d-\lambda^{TOT}))\\ &\quad = c_g'(x_g^{DAM*}(\text{proposed}))\\ &\quad = 2a\left(x_g^{DAM*}(\text{proposed})\right) + b.\end{aligned} \tag{41}$$

Since the cost function $c_g(x_g)$ is strictly convex, the claim is proved. □

D. Proof of Proposition 7.1

Proof: Squaring (15) and taking expectations, we obtain

$$\begin{aligned}&\mathbb{E}_\Xi[(\text{profit}_{r_{TOT}}^{\text{proposed}})^2]\\ &\quad = \mathbb{E}_\Xi[(\text{profit}_{r_{TOT}}^{\text{baseline}})^2]\\ &\qquad + 2\,x_{r_{TOT}}^{DAM}\,\mathbb{E}_\Xi[\text{profit}_{r_{TOT}}^{\text{baseline}}\,(\alpha - c_g'(d-\lambda^{TOT}))]\\ &\qquad + (x_{r_{TOT}}^{DAM})^2\,\mathbb{E}_\Xi\left[\left(\alpha - c_g'(d-\lambda^{TOT})\right)^2\right].\end{aligned} \tag{42}$$

Substituting (14) and (4) in (42), we get

$$\begin{aligned}&\mathbb{E}_\Xi[(\text{profit}_{r_{TOT}}^{\text{baseline}})^2] - \mathbb{E}_\Xi[(\text{profit}_{r_{TOT}}^{\text{proposed}})^2]\\ &\quad = 2\,x_{r_{TOT}}^{DAM}\,\mathbb{E}_\Xi[c_g'(d-\lambda^{TOT})^2\,\lambda^{TOT}]\\ &\qquad -2\,x_{r_{TOT}}^{DAM}\,\alpha\,\mathbb{E}_\Xi[c_g'(d-\lambda^{TOT})\,\lambda^{TOT}]\\ &\qquad - (x_{r_{TOT}}^{DAM})^2\ \text{Var}(c_g'(d-\lambda^{TOT})).\end{aligned} \tag{43}$$

Using the notation of Sections 5 and 7, let
$p = c_g'(d-\lambda^{TOT})$ and $\bar{\lambda} = x_{r_{TOT}}^{DAM} = \mathbb{E}_\Xi[\lambda^{TOT}]$.
The expectation value of renewable profit is the same for both markets. By applying the definition of covariance, the difference of variance between baseline and proposed market,

$$\begin{aligned}&\text{Var}(\text{profit}_{r_{TOT}}^{\text{baseline}}) - \text{Var}(\text{profit}_{r_{TOT}}^{\text{proposed}})\\ &\quad = 2\bar{\lambda}\ \text{Cov}(\lambda^{TOT}p,\ p) - \bar{\lambda}^2\ \text{Var}(p).\end{aligned} \tag{44}$$

Now, using the definition of covariance,

$$\text{Cov}(\lambda^{TOT}p, p) = \mathbb{E}_\Xi[\lambda^{TOT}p^2] - \mathbb{E}_\Xi[\lambda^{TOT}p]\mathbb{E}_\Xi[p]. \tag{45}$$

Let $\tilde{p} = p - \alpha$
Thus,

$$\mathbb{E}_\Xi[\lambda^{TOT}p^2] = \alpha^2\mathbb{E}_\Xi[\lambda^{TOT}] + 2\alpha\,\mathbb{E}_\Xi[\lambda^{TOT}\tilde{p}] + \mathbb{E}_\Xi[\lambda^{TOT}\tilde{p}^2], \tag{46}$$

$$\mathbb{E}_\Xi[\lambda^{TOT}p] = \mathbb{E}_\Xi[\lambda^{TOT}(\alpha+\tilde{p})] = \alpha\mathbb{E}_\Xi[\lambda^{TOT}] + \mathbb{E}_\Xi[\lambda^{TOT}\tilde{p}]. \tag{47}$$

Inserting (46) and (47) in (45) and using the definition of covariance, we get

$$\begin{aligned}&\text{Cov}(\lambda^{TOT}p, p)\\ &\quad = \alpha\ \text{Cov}(\lambda^{TOT}, p) + \text{Cov}(\lambda^{TOT}, (p-\alpha)^2) + \bar{\lambda}\,\text{Var}(p).\end{aligned} \tag{48}$$

Substituting (48) in (44), we obtain (24).

$$\begin{aligned}&\text{Var}(\text{profit}_{r_{TOT}}^{\text{baseline}}) - \text{Var}(\text{profit}_{r_{TOT}}^{\text{proposed}})\\ &\quad = \bar{\lambda}^2\,\text{Var}(p) + 2\bar{\lambda}\alpha\,\text{Cov}(\lambda^{TOT}, p)\\ &\qquad + 2\bar{\lambda}\,\text{Cov}(\lambda^{TOT}, (p-\alpha)^2).\end{aligned}$$

E. Proof of Proposition 7.2

Proof: From (25),

$$\mathrm{CTL}^{\mathrm{baseline}} = \mathrm{CTL}^{\mathrm{proposed}} + \bar{\lambda}(c_g'(d-\lambda^{TOT}) - \alpha).$$

Using the identity

$$\mathrm{Var}(X + aY) = \mathrm{Var}(X) + a^2\,\mathrm{Var}(Y) + 2a\,\mathrm{Cov}(X,Y),$$

we obtain

$$\begin{aligned}&\mathrm{Var}(\mathrm{CTL}^{\mathrm{baseline}})\\ &\quad= \mathrm{Var}(\mathrm{CTL}^{\mathrm{proposed}})\\ &\qquad+\bar{\lambda}^2\,\mathrm{Var}(c_g'(d-\lambda^{TOT})-\alpha)\\ &\qquad+2\bar{\lambda}\,\mathrm{Cov}(\mathrm{CTL}^{\mathrm{proposed}}, c_g'(d-\lambda^{TOT})-\alpha).\end{aligned}$$

Since α is a constant,

$$\mathrm{Var}(c_g'(d-\lambda^{TOT})-\alpha) = \mathrm{Var}(c_g'(d-\lambda^{TOT})),$$

$$\begin{aligned}&\mathrm{Cov}(\mathrm{CTL}^{\mathrm{proposed}}, c_g'(d-\lambda^{TOT})-\alpha)\\ &\quad= \mathrm{Cov}(\mathrm{CTL}^{\mathrm{proposed}}, c_g'(d-\lambda^{TOT})).\end{aligned}$$

Therefore, the difference in variances is

$$\begin{aligned}&\mathrm{Var}(\mathrm{CTL}^{\mathrm{baseline}}) - \mathrm{Var}(\mathrm{CTL}^{\mathrm{proposed}})\\ &\quad= \bar{\lambda}^2\,\mathrm{Var}(c_g'(d-\lambda^{TOT}))\\ &\qquad+2\bar{\lambda}\,\mathrm{Cov}(\mathrm{CTL}^{\mathrm{proposed}}, c_g'(d-\lambda^{TOT})).\end{aligned} \tag{49}$$

Using the notation of Appendix D, inserting (17) in (49), yields (27).

$$\begin{aligned}&\mathrm{Var}(\mathrm{CTL}^{\mathrm{baseline}}) - \mathrm{Var}(\mathrm{CTL}^{\mathrm{proposed}})\\ &\quad= \bar{\lambda}^2\,\mathrm{Var}(p) + 2\bar{\lambda}\,\mathrm{Cov}((d-\bar{d})p, p).\end{aligned}$$

Now, using the definition of covariance,

$$\mathrm{Cov}((d-\bar{d})p,p) = \mathbb{E}_\Xi[p^2(d-\bar{d})] - \alpha\mathbb{E}_\Xi[p(d-\bar{d})]. \tag{50}$$

Since $\mathbb{E}_\Xi[d-\bar{d}] = 0$,

$$\mathbb{E}_\Xi[p^2(d-\bar{d})] = 2\alpha\,\mathbb{E}_\Xi[\tilde{p}(d-\bar{d})] + \mathbb{E}_\Xi[\tilde{p}^2(d-\bar{d})], \tag{51}$$

$$\mathbb{E}_\Xi[p(d-\bar{d})] = \alpha\,\mathbb{E}_\Xi[d-\bar{d}] + \mathbb{E}_\Xi[\tilde{p}(d-\bar{d})] = \mathbb{E}_\Xi[\tilde{p}(d-\bar{d})]. \tag{52}$$

Substituting (51) and (52) in (50), we get

$$\begin{aligned}&\mathrm{Cov}((d-\bar{d})p,p)\\ &\quad= 2\alpha\,\mathbb{E}_\Xi[\tilde{p}(d-\bar{d})] + \mathbb{E}_\Xi[\tilde{p}^2(d-\bar{d})]\\ &\qquad-\alpha\,\mathbb{E}_\Xi[\tilde{p}(d-\bar{d})]\\ &\quad= \alpha\,\mathbb{E}_\Xi[\tilde{p}(d-\bar{d})] + \mathbb{E}[\tilde{p}^2(d-\bar{d})]\\ &\quad= \alpha\,\mathrm{Cov}(p,d) + \mathrm{Cov}(d,(p-\alpha)^2).\end{aligned} \tag{53}$$

Inserting (53) in (27), we obtain (26).

$$\begin{aligned}&\mathrm{Var}(\mathrm{CTL}^{\mathrm{baseline}}) - \mathrm{Var}(\mathrm{CTL}^{\mathrm{proposed}})\\ &\quad= \bar{\lambda}^2\,\mathrm{Var}(p) + 2\bar{\lambda}\alpha\,\mathrm{Cov}(d,p) + 2\bar{\lambda}\,\mathrm{Cov}(d,(p-\alpha)^2).\end{aligned}$$

Table 5
Hourly values of the three components of the variance difference and their aggregate contribution in Proposition 7.1 (all values $\times 10^{10}$, in $(\$/\mathrm{MWh})^2$)

Hour	Δ_1	Δ_2	Δ_3	$\Delta\mathrm{profit}_r$
0	2.839	-2.328	-0.176	0.335
1	3.110	-2.064	-0.083	0.963
2	3.347	-2.052	-0.016	1.279
3	4.261	-2.258	-0.624	1.379
4	5.169	-2.485	-2.950	-0.266
5	7.202	-3.072	-8.267	-4.137
6	11.02	-4.849	-52.94	-46.77
7	15.57	-5.488	-117.0	-106.9
8	14.92	-5.271	-126.5	-116.9
9	10.23	-3.776	-13.68	-7.226
10	9.854	-3.355	-3.887	2.612
11	10.34	-3.266	-1.844	5.230
12	16.22	-3.238	-0.755	12.23
13	30.23	-4.146	-6.126	19.96
14	25.17	-3.590	-5.985	15.60
15	361.8	-8.747	-329.8	23.25
16	230.5	-11.46	-207.4	11.64
17	177.2	-26.54	-52.12	98.54
18	193.6	-33.91	-87.75	71.94
19	1094	-52.40	-327.7	713.9
20	755.7	-36.74	-311.6	407.4
21	10.02	-6.079	-11.05	-7.109
22	3.078	-3.043	-0.975	-0.940
23	2.491	-0.206	0.246	2.531

F. Empirical Analysis Using ERCOT Data

For convenience, the three terms in (24) are denoted by

$$\begin{aligned}\Delta_1 &= \bar{\lambda}^2\,\mathrm{Var}(p),\\ \Delta_2 &= 2\bar{\lambda}\alpha\,\mathrm{Cov}(\lambda^{TOT}, p),\\ \Delta_3 &= 2\bar{\lambda}\,\mathrm{Cov}(\lambda^{TOT}, (p-\alpha)^2),\end{aligned}$$

The aggregate contribution is given by

$$\Delta\mathrm{profit}_r = \Delta_1 + \Delta_2 + \Delta_3.$$

Table 5 reports the hourly values of Δ_1, Δ_2, and Δ_3 from Electricity Reliability Council of Texas (2024a,b,c), averaged across the 366 days of 2024, together with their aggregate contribution $\Delta\mathrm{profit}_r$.
Analogously, for (26), we define

$$\begin{aligned}\Delta_4 &= 2\bar{\lambda}\alpha\,\mathrm{Cov}(d, p),\\ \Delta_5 &= 2\bar{\lambda}\,\mathrm{Cov}(d, (p-\alpha)^2),\end{aligned}$$

The aggregate contribution is given by

$$\Delta_{\mathrm{CTL}} = \Delta_1 + \Delta_4 + \Delta_5.$$

Table 6 reports the hourly values of Δ_1, Δ_4, and Δ_5 from Electricity Reliability Council of Texas (2024a,b,c,d), averaged across the 366 days of 2024, together with their aggregate contribution Δ_{CTL}.

Table 6
Hourly values of the three components of the variance difference in Proposition 7.2 and their aggregate contribution (all values $\times 10^{10}$, in $(\$/\mathrm{MWh})^2$)

Hour	Δ_1	Δ_4	Δ_5	Δ_{CTL}
0	2.839	1.162	1.880	5.8810
1	3.110	1.302	1.428	5.8400
2	3.347	1.382	2.503	7.2320
3	4.261	1.447	4.906	10.614
4	5.169	1.535	13.59	20.294
5	7.202	2.459	68.80	78.461
6	11.02	3.860	317.2	332.08
7	15.57	5.553	628.5	649.62
8	14.92	2.765	547.4	565.09
9	10.23	1.683	42.53	54.443
10	9.854	2.758	7.115	19.727
11	10.34	4.218	1.545	16.103
12	16.22	6.740	3.362	26.322
13	30.23	9.424	1.735	41.389
14	25.17	11.88	4.137	41.187
15	361.8	9.956	-214.2	157.56
16	230.5	8.400	-141.9	97.000
17	177.2	3.217	-8.674	171.74
18	193.6	11.13	-3.077	201.65
19	1094	84.10	1204	2382.1
20	755.7	40.00	512.9	1308.6
21	10.02	3.278	7.682	20.980
22	3.078	1.454	3.822	8.3540
23	2.491	1.284	1.923	5.6980