A Fair Division Approach to Humanitarian Logistics Incorporating Conditional Value-at-Risk*

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Abstract

Organization and efficiency of relief operations are vital following a major disaster, as well as the guarantee that all of the affected population will adequately have their basic needs met. However, in a post-disaster environment, uncertainty often impacts all aspects of the relief efforts. Placement of relief distribution centers, as well as public knowledge of these locations, is crucial to the speed and efficiency of relief efforts.

This research aims to develop a formulation to chose a set of distribution centers to open from a list of available facilities and to assign every member of the population to a distribution center. While developing these assignments, the costs to the affected population are considered in the form of travel costs to reach the assigned distribution center. Incorporation of these travel costs, a form of deprivation costs, minimizes the suffering of the population, and inclusion of ideas from fair division minimizes disparities in these costs to provide each member of the affected population with a fair level of service.

Further, the inclusion of a term inspired by conditional value-at-risk, or CVaR, into the formulation helps to further minimize potential disparities. Computational results for two datasets will be discussed to show the impact of including deprivation costs in this humanitarian logistics model. Additionally, theoretical results will show that optimal solutions to the formulation are guaranteed to be Pareto efficient.

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1 Introduction

In the wake of a disaster, coordination of relief efforts greatly influences both the time needed for the area to recover and the severity of the impact of the disaster. Disasters can have impacts in many ways, destroying personal property, damaging infrastructure, and injuring or killing those in the area [22]. Large scale disasters can affect millions; the Haitian earthquake in 2010 left over 1.5 million people homeless and affected over 3 million people in the area [24]. Fortunately, many groups and organizations ranging from governmental organizations such as FEMA in the United States to private organizations and faith-based groups are typically involved in relief efforts to mitigate the damage to the area and the suffering of the local population [20]. However, oftentimes relief efforts have been proven to be unfair, with some groups of the population receiving more aid than others.

These claims are a worldwide phenomenon, encompassing, for example, relief efforts following events as varied as Hurricane Katrina in the United States [8], the floods in Pakistan in both 2010 and 2014 [1] [21], the decade-long civil war in Nepal [11], and the Wenchuan earthquake in China in 2008 [25]. Claims range from unequal availability of temporary housing [8], to misappropriation of relief funds by political groups [1], to complete absence of relief for families affected by a disaster [21]. With 20.32% of people affected by the Wenchuan earthquake claiming that relief efforts were unfair, there is mounting evidence that unequal relief distribution impacts a substantial portion of populations affect by disasters [25]. Further, evidence has shown that the distribution of aid at all to an area may be highly political; in the United States, the two years with the highest number of declared disasters were both in years when the president was up for re-election, with the largest increase in declared disasters occurring in politically important states [20].

Additionally, logistics models for relief efforts are often based on commercial logistics models that focus mainly on operational costs and do not account for the suffering endured by the affected population [13]. The goal of this research is to incorporate the costs resulting from the suffering of the population into a fair division approach to locating distribution centers for supplies following a disaster so that no part of the affected population is neglected during the relief efforts.

Costs due to travel distance will be included in our formulation, as well as operational costs. These costs help to account for the population distribution and help to insure that the entire population will be adequately served. Walking costs serve as an approximation to deprivation costs and they are nonlinear, convex and monotonically increasing. Parameters are included to vary the weights on operational costs versus walking costs to see the effect higher walking costs has on the distribution of the relief centers.

Furthermore, incorporation of a term inspired by conditional value-at-risk, or CVaR, is used to create a distribution of relief centers that will minimize suffering due to walking costs for those who would otherwise incur the highest levels of walking costs among the population. The inclusion of the CVaR-inspired term serves to mitigate the effect of two potential sources of uncertainty: (i) exact values for deprivation costs are hard to establish at the high end, and the CVaR-inspired term increases the importance of these larger walking costs and gives a solution that better serves the people at greatest risk, and (ii) the population distribution may not be known exactly in the wake of a disaster, so it is necessary to ensure the people at greatest risk receive adequate service, in case their numbers are underestimated.

Background information is contained in Section 2. Our CVaR-inspired model for locating points of distribution can be found in Section 3. This model is generalized to other fair division problems in Section 4. Computational results are in Section 5, and Section 6 contains conclusions.

2 Background

2.1 Humanitarian Logistics

Humanitarian logistics, or the logistics surrounding post-disaster humanitarian relief efforts, first garnered attention following major maritime disasters in the 1970s [7]. Events are classified as disasters if the impacted area is incapable of organizing and managing the extent of relief efforts without outside assistance, and severe versions of disasters are labeled as catastrophes [12]. Disasters can be either natural disasters such as hurricanes, snowstorms, and tsunamis or made-man disasters such as the aftermath of conflicts.

There are four categories of humanitarian logistics: mitigation, preparedness, response, and recovery. The mitigation phase occurs prior to disasters and includes building of infrastructure to minimize impacts from storms. Preparedness also occurs before disaster strikes and includes the staging of supplies in order to facilitate distribution following the disaster. Both response and recovery follow the disaster, with response including the immediate efforts and recovery encompassing everything from debris removal and supply distribution to the reconstruction of buildings damaged in the disaster [9] [12].

As previously noted, humanitarian logistics models are frequently based on commercial logistics and do not account for the suffering of the affected population. The costs due to this suffering are known as deprivation costs and include costs due to time spent without access to needed supplies and travel costs and wait times to acquire supplies [13]. Data to model these costs is difficult to obtain, but deprivation costs can be reasonably expected to be nonlinear, convex, and monotonically increasing [13].

2.2 Fair Division

While fair division originally focused on income equality, the ideas in this field now extend to topics varying from settlement of border disputes to inheritance division [4] [18] [6]. The goal of fair division is to take a good or group of goods and create an allocation between the agents, those interested in obtaining the goods, that is deemed fair. Each agent assigns each good or fraction of a good a certain utility, where utility is additive and sums to 1 for all goods.

Many different ideas exist about how to determine a fair division, ranging from maximizing the minimum utility of agents to maximizing the total utility of the agents to optimizing the utilities based on the relative needs of the agents [5, 14]. Similarly, there are many ways to describe how fair an allocation is. For n agents, an allocation is called proportional if each agent has a utility of at least $1/n$, equitable if the utility for each agent is the same, and envy-free if a given agent cannot increase their utility by swapping assigned goods with another agent, i.e. no agent would prefer another's share [6]. Additionally, an allocation to a fair division problem is called Pareto efficient if there is no other allocation in which the utility of one agent is strictly greater than in the given allocation and the utilities of all other agents are at least as large [6]. No procedure can be guaranteed to always result in an allocation that is both Pareto optimal and envy-free.

Fair division problems can also involve the allocation of chores, in which case utility functions become disutility functions and the aim for each agent is to have a disutility of 0. For the allocation of chores, common approaches include minimizing the maximum disutility and minimizing the average disutility, which is comparable to minimizing the total disutility. The approach used in this paper will combine the ideas of these two approaches in order to guarantee that no single individual has a high level of disutility while also accounting for the disutilities of all agents. We will show that our procedure results in an allocation that is Pareto optimal. Minimizing the average disutility also results in a Pareto optimal allocation, and a minimax allocation that is Pareto optimal also exists.

2.3 Conditional Value-at-Risk

δ-Conditional value-at-risk, or δ-CVaR, is a risk measure that gives the conditional expectation of costs about δ-value-at-risk (δ-VaR) [2]. Furthermore, CVaR is a coherent risk measure; it is subadditive, monotonic increasing, translation invariant, and positive homogeneous [3] [19]. CVaR can be used for either continuous or discrete data; our focus is on discrete data and thus we use the definition for CVaR with discrete outcomes as described by Rockafellar and Uryasev [19]:

Definition 1. *The* CVaR *of a random variable $\zeta \in Z$ at the confidence level δ for a given decision variable x is defined as*

$$CVaR_{\delta}(Z) = \inf_{\eta \in \mathbb{R}} \left\{ \eta + \frac{1}{1-\delta} \sum_{\zeta \in Z} \mathbb{E}\left([\Psi(x,\zeta) - \eta]_{+} \right) \right\} \tag{1}$$

Note that this definition does not rely on VaR, and involves only linear terms.

While CVaR is traditionally used for stochastic problems, the formulation we developed is not stochastic. Rather than using δ-CVaR to calculate the expected cost for the most costly $1-\delta$ given scenarios, we use δ-CVaR to instead calculate the expected walking costs for the $1-\delta$ members of the population with the highest walking costs. This allows us to balance the minimization of the highest walking cost with the average walking costs, as previously described.

3 A Model for Locating Points of Distribution

The goal of this research is to place points of distribution, or PODs, from which to distribute relief supplies within a region given the population distribution and road network. This placement problem assumes that the road network is fully known and can be used either pre- or post-disaster to locate PODs. If used pre-disaster, it is recommended to develop a level of damage to the road network that could reasonably be expected to be an outcome of the disaster based on damage reports from prior similar disasters in the area. The PODs to be used will be chosen from a list of potential POD locations that are chosen based on suitability. These should typically be large, well-known buildings that are accessible to all members of the population. For example, a public school building is a reasonable choice to include in the list of potential POD locations, but a private school run by a religious organization is not as individuals who are not members of the religion may feel excluded.

The PODs each have a known capacity and a known operating cost. Typically, operating costs will be lower per person at facilities that can accommodate a larger number of people and will depend on the location of the facility; facilities closer to the boundary of the affected region can be expected to have lower operating costs as the location facilitates the delivery of supplies from outside areas.

A discrete population distribution is used in the problem formulation in order to estimate population densities across the region. This is accomplished through the location of population centers, each with a known location and population total. The population is allowed to vary across these population centers to better estimate the true population distribution. Each population center will be assigned to a POD or to multiple PODs as needed, with deprivation costs due to travel distance and difficulty incorporated into the formulation. These deprivation costs are developed outside of the main formulation and should be a convex monotonic increasing function of the distance traveled. Roads with significant damage may be assigned higher costs than roads of the same length with no damage.

The formulation used is based on standard capacitated facility location models, with modifications to include walking costs and ideas from fair division in an effort to eliminate disparities in deprivation costs across the population of the region. The goal of providing fair service to all in the affected population regardless of demographic features is the motivation behind incorporating these ideas from fair division. Here, the allocation of PODs was treated as an allocation of chores; every POD is assumed to provide the same level of service to each individual, but with varying walking costs based on distance between the individual's location and the POD which acts as a disutility function for each individual of the population. As such, the objective includes both the average walking cost for the population and the δ-CVaR of the walking cost across the population in order to minimize disutility, particularly for those who have the highest walking costs. For example, if $\delta = 0.9$, the 0.9-CVaR term represents the average cost for the 10% of the population with the highest walking costs. Here, δ is treated as a parameter. Note that δ-CVaR is not being used in the traditional sense; there is no uncertainty here, with the walking cost for each individual to travel to every POD individually is known. Additionally for this formulation, the sets are defined as

$I :=$ the set of population centers;
$J :=$ the set of potential POD locations;

the variables are defined as

$x_j :=$ 1 if POD j is in use; 0 otherwise;
$y_{ij} :=$ the percentage of people at population center i who use POD j;
$\eta :=$ δ-VaR for optimal solutions;
$\Theta :=$ δ-CVaR for optimal solutions;
$v_{ij} :=$ the non-negative difference between the walking cost z_{ij} and the value of δ-VaR, η;

and the parameters are

$z_{ij} :=$ the walking cost incurred for a person in population center i who uses POD j;
$P_i :=$ the population at population center i;
$O_j :=$ the operational cost for using POD j;
$C_j :=$ the operating capacity, in people, of POD j;
$\alpha :=$ weight for δ-CVaR;
$\beta :=$ weight for the average walking cost.

This gives the following formulation:

$$\min_{\eta,\Theta,v,x,y} \sum_j O_j x_j + \alpha\Theta + \frac{\beta}{\sum_i P_i} \sum_i \sum_j P_i y_{ij} z_{ij} \tag{2a}$$

subject to:

$$\sum_{j \in J} y_{ij} \geq 1, \qquad \forall i \in I \tag{2b}$$
$$y_{ij} \leq x_j \qquad \forall i \in I, j \in J \tag{2c}$$
$$\sum_{i \in I} P_i y_{ij} \leq C_j, \qquad \forall j \in J \tag{2d}$$
$$y_{ij} \geq 0, \qquad \forall i \in I, j \in J \tag{2e}$$
$$\Theta \geq \eta + \frac{1}{1-\delta} \sum_{i \in I} \left(\frac{P_i}{\sum_{k \in I} P_k} \sum_{j \in J} y_{ij} v_{ij} \right) \tag{2f}$$
$$v_{ij} \geq z_{ij} - \eta, \qquad \forall i \in I, j \in J \tag{2g}$$
$$v_{ij} \geq 0, \qquad \forall i \in I, j \in J \tag{2h}$$
$$x_j \in \{0,1\}, \qquad \forall j \in J \tag{2i}$$
$$\eta \in \mathbb{R}, \tag{2j}$$

The objective (2a) of this mixed integer nonlinear program (MINLP) includes terms for the operational costs, the δ-CVaR of walking costs across the population, and the average walking cost for the population. Constraints (2b), (2c), and (2d) all follow from standard capacitated facility location problems guaranteeing that every population center is fully served, individuals are not sent to PODs that are inoperable, and that the capacity limit for each POD is enforced, respectively. Constraints (2f), (2g), and (2h) are used for the calculation of δ-CVaR, as described in [19] and [17]. Note that this formulation does not restrict the total disutility for each individual of the population to be 1. In reality, we would like to restrict $P_i y_{ij}$ to be integer, meaning a whole number of individuals should be assigned to each POD; otherwise at least one individual would be assigned to multiple PODs, equating to the individual receiving only part of their supplies from each location and having to travel to multiple locations to receive all of the needed supplies. However, in practice this constraint was not included in the formulation, but was always realized empirically.

4 Pareto Efficiency of a Fair Division Scheme Based on CVaR

Let $\mathcal{N} = \{1, \ldots, n\}$ denote the collection of agents and let $\mathcal{S}$ denote the set of feasible allocations of goods. Let $u_i(s) \geq 0$ denote the disutility of agent i for allocation $s \in \mathcal{S}$ for each $i \in \mathcal{N}$. Minimizing a weighted average disutility would require solving the problem

$$\min_{s \in \mathcal{S}} \quad \sum_{i \in \mathcal{N}} \gamma_i u_i(s) \tag{3}$$

for given positive weights γ_i, $i \in \mathcal{N}$. A weighted minimax allocation is a solution to the problem

$$\min_{s \in \mathcal{S}} \quad \max_{i \in \mathcal{N}} \mu_i u_i(s) \tag{4}$$

for an appropriate set of weights $\mu > 0$. We propose a combination of these approaches. In particular, we select a subset $\bar{\mathcal{N}} \subseteq \mathcal{N}$ and a proportion δ and require that the allocation should consider the average performance over the worst $1 - \delta$ of the elements of $\bar{\mathcal{N}}$. Our proposed allocation is a solution of the optimization problem

$$\min_{s \in \mathcal{S}, \eta \geq 0} \quad \sum_{i \in \mathcal{N}} \gamma_i u_i(s) + \eta + \frac{1}{1-\delta} \sum_{k \in \bar{\mathcal{N}}} [\mu_k u_k(s) - \eta]_+ \tag{5}$$

for positive weights γ and μ. If $\bar{\mathcal{N}} = \emptyset$ then problem (5) is equivalent to (3). Problem (4) can be recovered from (5) by setting $\bar{\mathcal{N}} = \mathcal{N}$ and taking the limit as $\gamma \to 0$ and $\delta \to 1$. Any solution to (5) is Pareto efficient, as we now prove.

Theorem 1. *Let $s^* \in \mathcal{S}$, $\eta^* \geq 0$ be an optimal solution to (5). The allocation s^* is Pareto efficient.*

Proof. Assume there exists another allocation $\bar{s} \in \mathcal{S}$ satisfying:

$$\begin{array}{rcll} u_i(\bar{s}) & \leq & u_i(s^*) & \forall i \in \mathcal{N} \\ u_l(\bar{s}) & < & u_l(s^*) & \text{for at least one } l \in \mathcal{N}. \end{array}$$

Since $\gamma > 0$ and $\mu > 0$, we have

$$\begin{aligned} &\textstyle\sum_{i \in \mathcal{N}} \gamma_i u_i(\bar{s}) + \eta^* + \frac{1}{1-\delta} \sum_{k \in \bar{\mathcal{N}}} [\mu_k u_k(\bar{s}) - \eta^*]_+ \\ &\qquad < \textstyle\sum_{i \in \mathcal{N}} \gamma_i u_i(s^*) + \eta^* + \frac{1}{1-\delta} \sum_{k \in \bar{\mathcal{N}}} [\mu_k u_k(s^*) - \eta^*]_+ \end{aligned}$$

so s^*, η^* is not an optimal solution to (5). □

In our setting, we have an inhomogeous fair division problem. The set of agents $\mathcal{N}$ is comprised of two groups. The obvious agents are the individuals of the population with disutilities given by the walking cost for their assigned POD; these agents comprise the set $\bar{\mathcal{N}}$. Further, we also have a less obvious agent. This agent can be described as a single organizational agent whose disutility is given by the total operational cost for all the PODs in use. The set $\mathcal{S}$ of "allocations of goods" consists of all feasible choices for the locations of the PODs together with a corresponding feasible allocation of individuals in the population to PODs.

With this list of agents, we cannot speak to properties such as proportionality, equitability, or envy-freeness; however, we can make a determination about Pareto efficiency for this model. Indeed, any optimal solution to the problem (2) will be Pareto efficient, as a consequence of Theorem 1:

Corollary 1. *Let $(\eta^*, \Theta^*, \nu^*, x^*, y^*)$ be an optimal solution to (2). The locations of the PODs given by x^* together with the allocation of individuals to PODs given by y^* is Pareto efficient.*

Proof. The disutility for the organizational agent is $\sum_{j \in J} O_j x_j$, a term that appears in the objective (2a). The total walking cost is the sum of the disutilities of the individuals in the population and is given by the $\sum_{i \in I} \sum_{j \in J} P_i y_{ij} z_{ij}$ term in the objective (2a). The constraints (2f), (2g), and (2h) ensure that the term Θ in the objective (2a) captures exactly the CVaR value over the individuals in the population. The remaining constraints define the set of feasible assignments, $\mathscr{S}$. Thus, problem (2) is in the same form as (5) and the result follows from Theorem 1. □

5 Computational Results

In this section, we will discuss computational results for two datasets and a wide variety of parameter choices. The first dataset is representative of a large town or small city and includes the rural areas surrounding the city and the second is data from a large city and does not include surrounding areas. For each dataset, four different vales of δ are examined with at least 16 α, β pairs used for each choice of δ, resulting in over 150 different optimization problems.

5.1 Solution Technique

As previously mentioned, the walking costs are not calculated as a part of the formulation and should be calculated separately and then treated as a parameter. This reveals what the highest possible walking cost for any person can be; obviously walking costs cannot be negative and are bounded below by zero. Since upper and lower bounds on the walking costs are known, upper and lower bounds on η, the value of δ-VaR, are also known.

This allows the Golden Section search to be used to solve this formulation; η can be treated as a parameter within formulation (2) and the Golden Section search can be used to determine the value of η by minimizing the value of formulation (2) as a function of η. Since η and z_{ij} are all parameters, v_{ij} can also be treated as parameters as they are simply the nonnegative difference between z_{ij} and η. Thus, the only variables are x_j, y_{ij}, and Θ. Further, this reduces the problem to a mixed integer linear program (MILP) from a MINLP; since v_{ij} is no longer a variable, constraint (2f) is now linear. By decreasing the number of variables and modifying the problem to be linear, we drastically decrease computational time. For more details on this method and this formulation in general, please see [10].

5.2 Medium Dataset

The first dataset examined was a medium sized generated dataset. The road network and population distribution were simulated to mimic an area with a small city and surrounding rural areas; this data was provided by Loggins [16]. In total, the area has a population of 19,000 people, spread evenly across 76 population centers as seen in Figure 1.

Since no information about available infrastructure was available other than the road network, a randomized method was used to designate various intersections as potential POD locations. For

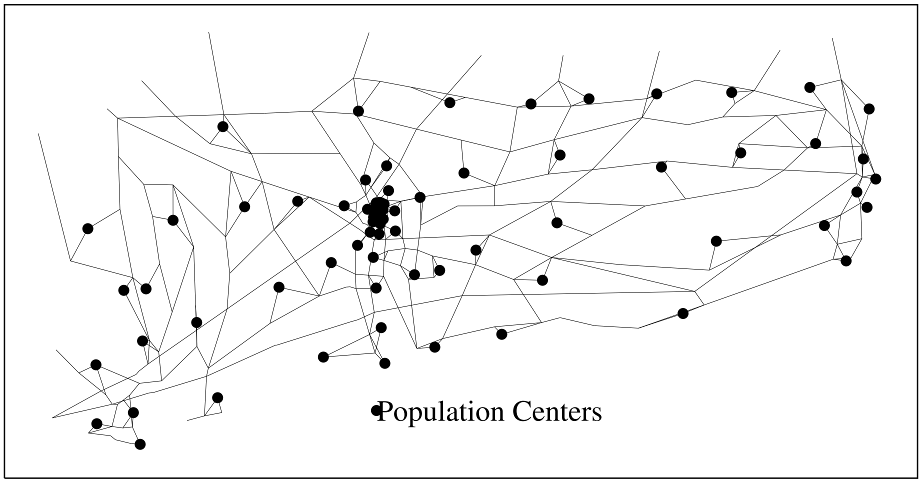

Figure 1: Population center locations

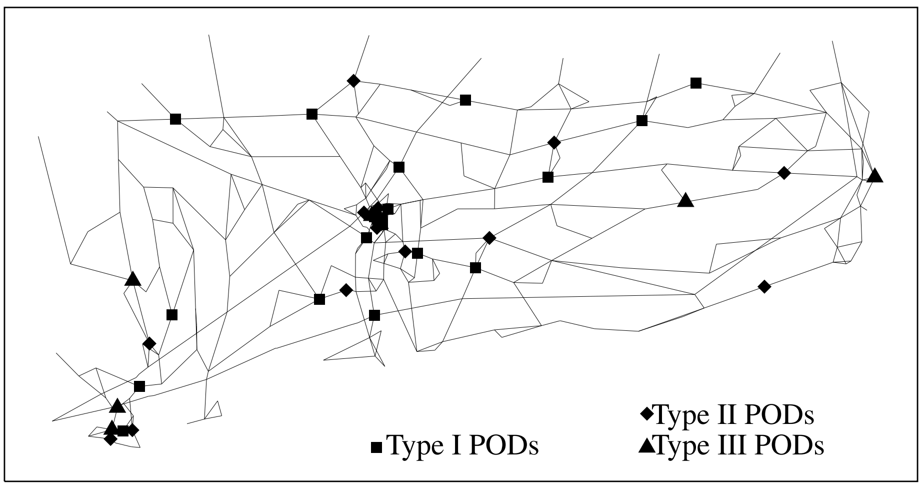

Figure 2: Potential POD locations

more details on this randomized, please refer to [10]. In total, 42 different potential POD locations were chosen, with a total capacity of 39,000. These potential PODs were divided into three groups based on assigned capacities. PODs with a capacity of 2000 people were designated at Type I PODs, a capacity of 1000 as Type II PODS, and a capacity of 500 as Type III PODs. Of the 42 PODs, 7 were Type I, 15 were Type II, and 20 were Type III. This ratio of capacities is consistent with the ratios used in standard relief operations [15]. These potential PODs can be seen in Figure 2.

Operational costs for PODs were based on information about the capacity and location of each POD. In general, larger PODs have a lower cost per person than the smaller PODs, and PODs closer to the boundary of the region have lower costs than PODs of the same size that are located near the center of the region. Location information was used in order to indirectly include information about the cost of delivering supplies to the POD.

5.2.1 Walking Cost Calculations

A shortest path problem was solved for each POD and population center pair to use as a basis for the walking costs. For this dataset, damage to the road network was included by removing 6 arcs from the network, as detailed in [10]. This simulated a mild level of damage to the area.

A three-part piecewise linear function of distance was used to calculate walking costs. Threshold distances of 5 miles and 10 miles were chosen. Distances under 5 miles had the lowest walking cost per mile, with the cost per mile increasing as each threshold was crossed. This piecewise linear function was designed to mimic the expected general trends of the true walking cost function, a function expected to be monotonic increasing, convex, and nonlinear.

5.2.2 Results

For this dataset, four different values of δ were examined: $\delta = 0.987, 0.95, 0.9, 0.8$. Here each population center had the same population, and every POD had a capacity which was an integer multiple of the population at each population center. As a result, population centers were not divided and every person at a given population center was sent to the same POD. As a result, $\delta = 0.987$ was analogous to looking at the walking cost for the individuals at the population center with the highest walking cost.

All computational results were obtained from ILOG AMPL 11.010, version 20080219, using CPLEX 11.0.1 for solution of the formulation in (5.2) and Python 3 to script the Golden Section search used to minimize η on a 1.8 GHz Intel Atom D525 dual-core processor with 1 MB L2 cache and 4 GB DDR3 RAM running Arch Linux (Linux kernel version 3.9.3-1). The average computation time was 11 minutes and 2.39 seconds across all values of α, β, and δ, with $\delta = 0.8$ having the lowest average computational time, less than 5 minutes [10].

For each value of δ, we will report on each of the three terms in the objective: the operational costs, $\sum_j O_j x_j$; the average walking cost, $\frac{\beta}{\sum_i P_i} \sum_i \sum_j P_i y_{ij} z_{ij}$; and the value of δ-CVaR, Θ. Additionally, we will provide information about the number and types of PODs chosen, and whether these PODs are operating at full capacity. A complete analysis of results can be found in [10].

For $\delta = 0.8$, there were a total of seven distinct solutions across 20 choices of α and β, ranging from a solution that chose 5 Type I PODs, 9 Type II PODs, and no Type III PODs with an average walking cost of 7.2 and operational costs of 118100, to a solution that chose 4 Type I PODs, 9 Type II PODs, and 9 Type III PODs with an average walking cost of 2.7 and operational costs of 152300. The first of these solutions only provided enough PODs to meet capacity requirements while the last had 6 PODs that were underutilized. The values of 0.8-CVaR ranged from 8.6944 to 22.1151. The minimum value of 0.8-CVaR for this dataset was 8.0313 and was not reached in any of the solutions.

For $\delta = 0.9$, there were a total of six distinct solutions across 20 choices of α and β, ranging from a solution that chose 4 Type I PODs, 10 Type II PODs, and 2 Type III PODs with an average walking cost of 4.9 and operational costs of 125500, to a solution that chose 5 Type I PODs, 9 Type II PODs, and 10 Type III PODs with an average walking cost of 2.6 and operational costs of 167100. The first of these solutions only provided enough PODs to meet capacity requirements

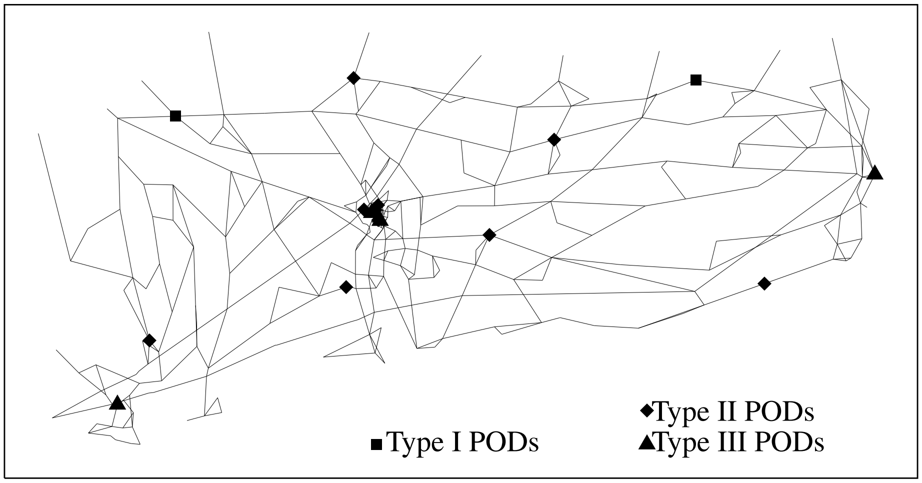

Figure 3: PODs chosen for 0.9-CVaR with the smallest scaling

while the last had 8 PODs that were underutilized. The values of 0.9-CVaR ranged from 8.6944 to 19.3363. The minimum value of 0.9-CVaR for this dataset was 11.6560 and was reached in one of the distinct solutions.

For $\delta = 0.95$, there were a total of four distinct solutions across 20 choices of α and β, ranging from a solution that chose 4 Type I PODs, 10 Type II PODs, and 2 Type III PODs with an average walking cost of 4.9 and operational costs of 125500, to a solution that chose 4 Type I PODs, 10 Type II PODs, and 5 Type III PODs with an average walking cost of 3.1 and operational costs of 139500. The first of these solutions only provided enough PODs to meet capacity requirements while the last had 3 PODs that were underutilized. The values of 0.95-CVaR ranged from 21.3958 to 16.2097. The minimum value of 0.95-CVaR for this dataset was 16.2097 and was reached in two of the distinct solutions. Additionally, this value of 0.95-CVaR was also reached for formulations with $\delta = 0.9$ and for formulations with $\delta = 0.8$.

For $\delta = 0.987$, there were a total of six distinct solutions across 16 choices of α and β, ranging from a solution that chose 5 Type I PODs, 10 Type II PODs, and no Type III PODs with an average walking cost of 5.3 and operational costs of 126600, to a solution that chose 4 Type I PODs, 10 Type II PODs, and 3 Type III PODs with an average walking cost of 4.2 and operational costs of 131000. The first of these solutions included enough PODs that 3 PODs were underutilized while the last had 2 PODs that were underutilized. The values of 0.987-CVaR ranged from 20.8608 to 22.0040. The minimum value of 0.987-CVaR for this dataset was 20.8608 was reached in the solutions of all but the lowest two scalings of the 0.987-CVaR formulation, and was reached in the formulations for all other values of δ.

Images of the results for $\delta = 0.9$ for the smallest values of α and β and for the largest values of α and β can be seen in Figures 3 and 4. These figures illustrate the move away from the Type I PODs to the Type II and III PODs as well as an increase in the total number of PODs. This shift is particularly evident in the rural areas, where the total number of PODs increases drastically. More details about the specific values for each formulation as well as images of all results can be found in [10].

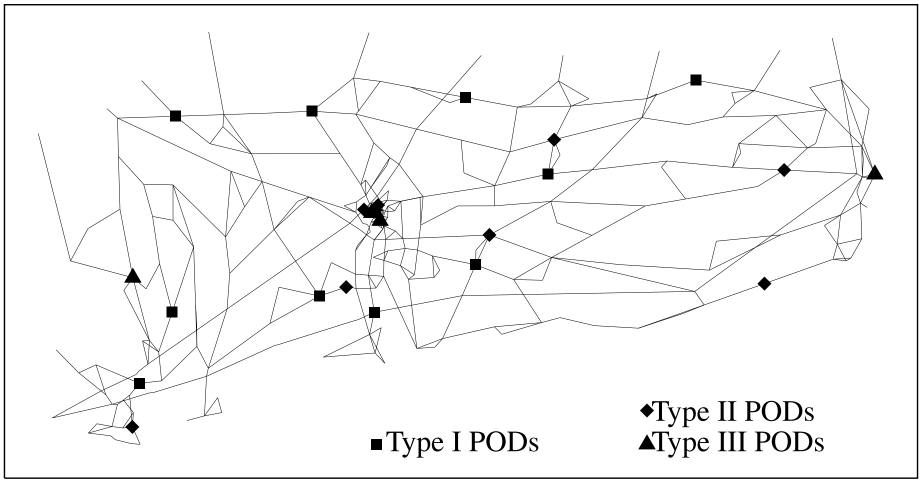

Figure 4: PODs chosen for 0.9-CVaR with the largest scaling

5.3 Large Dataset

The second dataset was a larger dataset consisting of the Mid-City, Uptown, Lakeview, and Gentilly areas of New Orleans, Louisiana, USA. The road network used is the New Orleans road network as available from the TransCAD Transportation Planning software. Census data at the zip code level was used to locate population centers. Each zip code was divided into two or three areas with each area assigned its own population center, with the total population for each zip code split evenly between the population centers. Total, there were 46 population centers with a total population of 454375. Unlike the smaller dataset, population centers could have different populations so fewer population centers could be used while still accurately modeling the population distribution. These population centers can be seen in Figure 5.

Since information about buildings in the area is available, public school buildings were used to develop this list of potential PODs. The locations of public schools are widely known among the local population and are neutral locations to use as PODs. In addition, the Mercedes-Benz Superdome, formerly the Louisiana Superdome, and the Ernest N. Morial Convention Center were also included in the list of potential PODs due to the heavy utilization of these buildings in the aftermath of Hurricane Katrina in 2005. Capacities at the school buildings were based on the student body size for the school, and the capacities of the additional two PODs were based off of the level of utilization following Hurricane Katrina. The capacities of these potential PODs varied from 2,000 to 40,000, and were broken into four groups. Type I PODs had capacities ranging from 13,000 to 19,000; Type II PODs from 9,000 to 12,600; Type III from 2,000 to 8,600. The largest PODs, with capacities ranging from 34,000 to 40,000 were classified as super-PODs. In total, there were 76 potential PODs, with 3 super-PODs, 20 Type I PODs, 26 Type II PODs, and 25 Type III PODs. These potential PODs can be seen in Figure 6

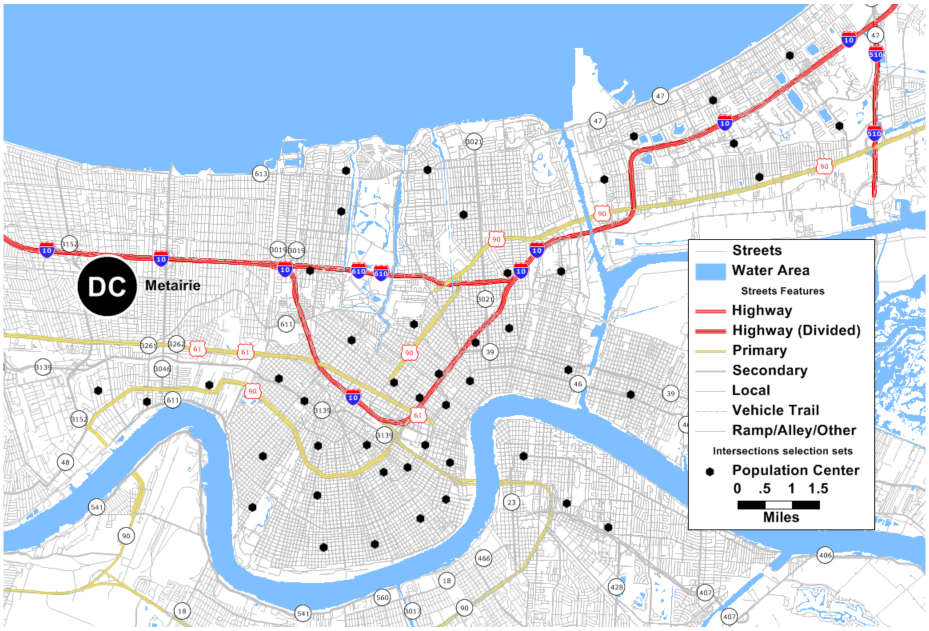

Figure 5: Population centers for Hurricane Katrina data

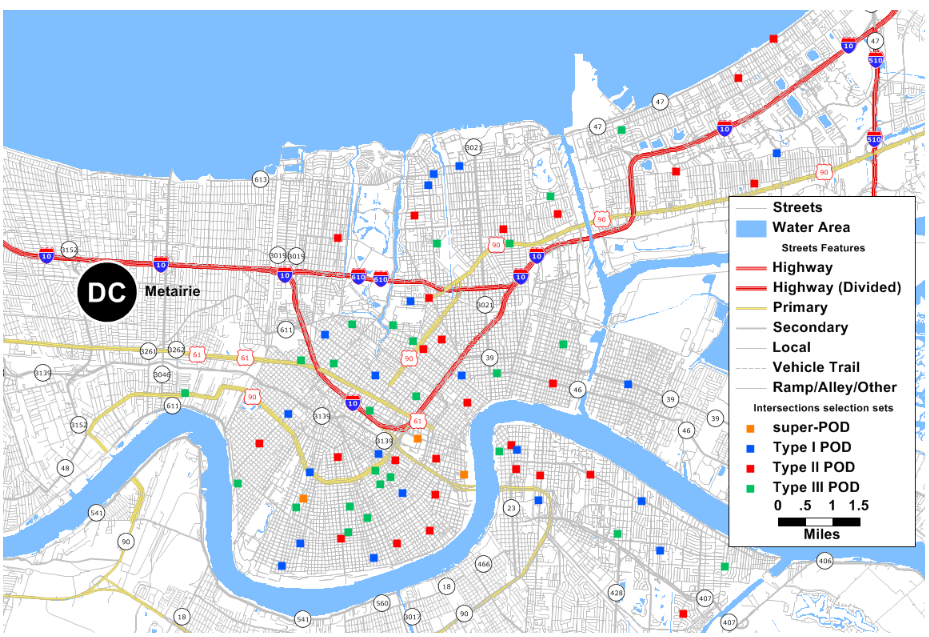

Figure 6: PODs for Hurricane Katrina data

5.3.1 Walking Cost Calculations

For this dataset, we assumed a high level of damage to the road network; the damage level used was based off of the maximum level of flooding following Hurricane Katrina, as reported by [23]. For each arc of the road network, a cost was calculated based off of the length of the arc and the level of flooding on the arc. Costs were assumed to be an exponential function of the flood level to represent the increasing difficulty of travel as the flood increases.

Once each arc was assigned a cost based on its length and flood level, the shortest path from each population center to each POD was defined using these assigned costs instead of simply the distance. Thus the 'shortest path' may not be the path with the shortest distance and may instead be a longer route that avoids severely flooded arcs. Due to the extensive level of damage to the network, walking costs increased drastically, and as such walking costs were only calculated for the 30 PODs closest to each population center and large walking costs were used for the remaining population center and POD pairs.

5.3.2 Results

Results were obtained from CPLEX 12.6.1.0 on an Intel(R) Core(TM)2 Quad CPU Q9550 running at 2.83 GHz with 6 MB of L2 cache and 4 GB DDR3 RAM running Arch Linux (Linux kernel version 3.10.17). Again, four values of δ were used in the formulation, $\delta = 0.8, 0.9, 0.95, 0.99$. For this dataset, 30 different values of α and β were used for each value of δ.

For each value of δ, we will report on each of the three terms in the objective: the operational costs, $\sum_j O_j x_j$; the average walking cost, $\frac{\beta}{\sum_i P_i} \sum_i \sum_j P_i y_{ij} z_{ij}$; and the value of δ-CVaR, Θ. Additionally, we will provide information about the number and types of PODs chosen, and whether these PODs are operating at full capacity. A complete analysis of results can be found in [10].

For $\delta = 0.8$, there were a total of eight distinct solutions across 30 choices of α and β, ranging from a solution that chose 3 super-PODs, 17 Type I PODs, 6 Type II PODs, and no Type III PODs with an average walking cost of 4.9 and operational costs of 3428700, to a solution that chose 3 super-PODs, 15 Type I PODs, 7 Type II PODs, and 4 Type III PODs with an average walking cost of 2.9 and operational costs of 3512850. The first of these solutions provided enough PODs that 2 PODs were underutilized while the last had 4 PODs that were underutilized. The values of 0.8-CVaR ranged from 5.9160 to 9.9937. The minimum value of 0.8-CVaR for this dataset was 5.6074 and was not reached in any of the solutions.

For $\delta = 0.9$, there were a total of eight distinct solutions across 30 choices of α and β, ranging from a solution that chose 3 super-PODs, 17 Type I PODs, 6 Type II PODs, and no Type III PODs with an average walking cost of 4.7 and operational costs of 3428700, to a solution that chose 3 super-PODs, 16 Type I PODs, 7 Type II PODs, and 2 Type III PODs with an average walking cost of 3.0 and operational costs of 3490850. The first of these solutions provided enough PODs that 2 PODs were underutilized while the last had 3 PODs that were underutilized. The values of 0.9-CVaR ranged from 6.8898 to 10.8689. The minimum value of 0.9-CVaR for this dataset was 6.7003 and was not reached in any of the distinct solutions.

For $\delta = 0.95$, there were a total of six distinct solutions across 30 choices of α and β, ranging

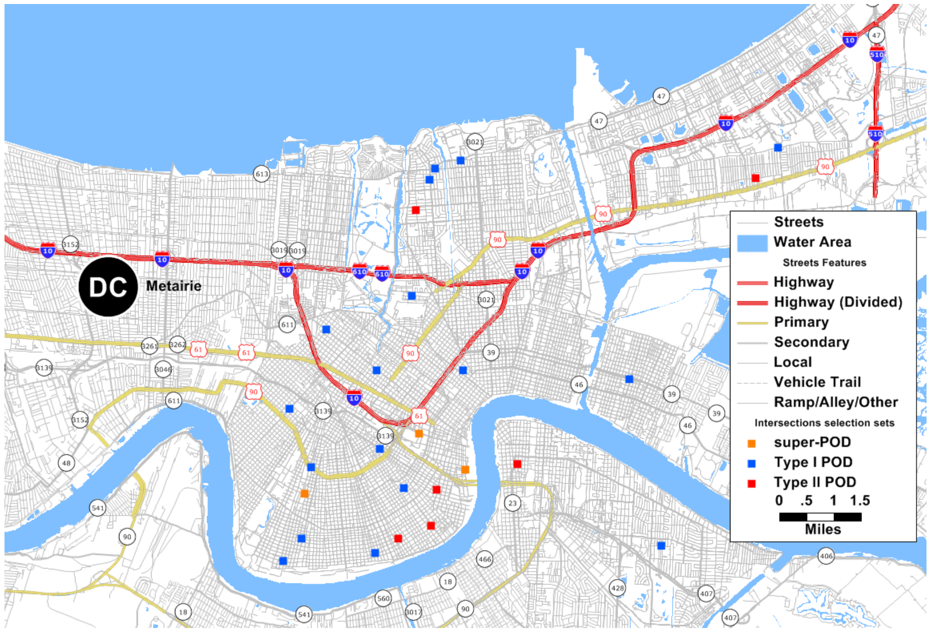

Figure 7: PODs chosen for 0.9-CVaR for the smallest scaling

from a solution that chose 3 super-PODs, 17 Type I PODs, 6 Type II PODs, and no Type III PODs with an average walking cost of 4.7 and operational costs of 3428700, to a solution that chose 3 super-PODs, 16 Type I PODs, 6 Type II PODs, and 3 Type III PODs with an average walking cost of 3.1 and operational costs of 3476650. The first of these solutions provided enough PODs that 2 PODs were underutilized while the last had 4 PODs that were underutilized. The values of 0.95-CVaR ranged from 7.4325 to 11.4567. The minimum value of 0.95-CVaR for this dataset was 7.3607 and was not reached in any of the distinct solutions.

For $\delta = 0.99$, there were a total of seven distinct solutions across 30 choices of α and β, ranging from a solution that chose 3 super-PODs, 17 Type I PODs, 6 Type II PODs, and no Type III PODs with an average walking cost of 4.6 and operational costs of 3428700, to a solution that chose 3 super-PODs, 17 Type I PODs, 6 Type II PODs, and 1 Type III PODs with an average walking cost of 3.2 and operational costs of 3645250. The first of these solutions included enough PODs that 2 PODs were underutilized while the last had 3 PODs that were underutilized. The values of 0.99-CVaR ranged from 7.9581 to 12.7422. The minimum value of 0.99-CVaR for this dataset was 7.9581 and was reached in one of the distinct solutions of the 0.99-CVaR formulation, and was reached in the formulations for all values of δ except $\delta = 0.8$.

The results of from the 0.9-CVaR formulations with the smallest and largest values of α and β can be seen in Figures 7 and 8, respectively. More details about the specific values for each formulation as well as images of all results can be found in [10].

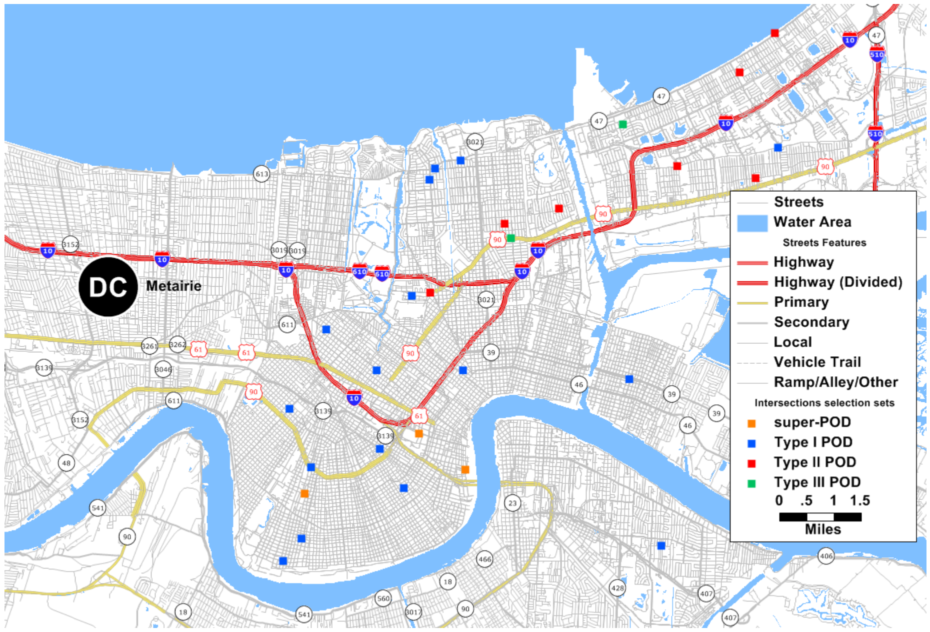

Figure 8: PODs chosen for 0.9-CVaR for the largest scaling

5.4 General Trends

For nearly all choices of δ, α, and β, the optimal solution included using more PODs than is necessitated by the population total for the area, showing that walking costs do influence the optimal solutions. Obviously this resulted in some PODs operating below capacity. This inclusion of additional PODs shows that simply including minimum service level constraints is not sufficient to serve the affected population. Additionally, this excess capacity does help to provide flexibility in case population estimates were inaccurate.

For each value of δ, the total number of PODs chosen increased as α and β increased. Further, as α and β increased, the number of larger PODs chosen decreased while more smaller PODs were included in the solution. This shows that using more small sized PODs is more beneficial to the population than simply using fewer of the larger PODs. Using mostly small PODs allows the PODs to be more widely distributed while resulting in fewer PODs that are underutilized.

6 Conclusions and Future Work

We see from these results, that ignoring deprivation costs in the formulation of disaster relief problems will result in solutions that do not provide the best level of service for the affected population. Additionally, including deprivation costs may also increase the flexibility of the solutions; here we saw solutions with additional network capacity which provides flexibility if the population distribution is not accurate.

Furthermore, including ideas from fair division helps to alleviate disparities in service levels for the affected population by taking the costs to every individual into account, and not simply the costs to a subset of the population. While we assumed that every individual had the same level of need, the deprivation costs can be modified to account for demographics such as age of the population and disability status.

Future work includes the determination and incorporation of other types of deprivation costs into disaster relief models. Additionally, further research is need to accurately determine what the values of these deprivation costs should be, as well as how they are affected by the characteristics of the population and the level of damage to the network.

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