Disruptions in Interdependent Infrastructure Systems: A Network Flows Approach[1]

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Abstract — Modern society depends on the operations of civil infrastructure systems, such as transportation, energy, telecommunications and water. Clearly, disruption of any of these systems would present a significant detriment to daily living. However, these systems have become so interconnected, one relying on another, that disruption of one may lead to disruptions in all. The focus of this research is on developing techniques which can be used to respond to events that have the capability to impact interdependent infrastructure systems. As discussed in the paper, infrastructure interdependencies occur when, due to either geographical proximity or shared operations, an impact on one infrastructure system affects one or more other infrastructure systems. The approach is to model the salient elements of these systems and provide decision makers with a means to manipulate the set of models, i.e. a decision support system.

Definitions of five types of interdependency identified during the research are presented and incorporated into three network flows mathematical representations. The first representation

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describes each system during normal operations. The second provides support to the managers of the individual systems and to emergency response officials in assessing the impact of a disruption and determining if service can be provided without extensive restoration operations. The third model shows the impact of a disruption when interdependencies among infrastructures are considered and supports strategy development and decision making during restoration. An illustrative example of the models is presented. The paper concludes with a discussion of accomplishments and opportunities for future work.

Index Terms — Civil Infrastructure Systems, Decision Support Systems, Emergency Management, Mathematical Programming, Networks

1. Introduction

Modern society relies on the operations of a set of human-built systems and their processes. The set of systems which is investigated by this research is referred to as civil infrastructure systems. These systems are typically considered to be transportation (including roads, bridges, water and rail); energy (including electric power, gas and liquid fuels); telecommunications (including telegraph, telephone, wireless and internet/digital); and finally, water (including wastewater facilities and water supplies). All civil infrastructures systems rely on a constructed system in order to provide services, such as power delivery, voice and data transmission. Each system's components can only be used to support services of their respective group (communications lines cannot be used for energy transmission and vice versa; water system pipelines are not readily available for energy products such as gas or fuel). This set of systems has been included in the broader set of critical infrastructures defined by the President's Council on Critical Infrastructure Protection (PCCIP) [1].

When an event occurs that may cause disruptions to more than one infrastructure system or is considered to be beyond the management capability of normal staff, emergency response organizations are activated. Emergency Response Organizations (EROs) exist not only at the federal, state, county or city level, but within organizations responsible for operation of the infrastructure systems [2,3]. Immediately after the September 11, 2001 attacks in New York City, many emergency response organizations were activated. For New York City, the ERO is the Office of Emergency Management (NYCOEM); at the state level, it is the Emergency Management Office (NYSEMO); within Consolidated Edison (the principal supplier of power), it is the Corporate Emergency Response Center; for Verizon, a telecommunications provider, it is the Emergency Command Center. No matter the name, each of these emergency response organizations is established for the same basic reasons: to set priorities, coordinate response efforts, collect information and keep informed all relevant parties, both within and external to the organization [4]. For example, following the 9/11 attacks, ConEd established initial response priorities for crews and kept NYCOEM informed. As NYCOEM became aware of needs, requests were made to responsible agencies or companies. Additionally, coordination of resources was made at NYCOEM as they were made aware of the resources each agency or company had available for response and restoration of services. When a priority was established by federal, state or city government officials, it was the responsibility of NYCOEM to make this priority clear to all member agencies.

As a result of our case analyses, we chose to focus on supporting the EROs in the organizations responsible for managing civil infrastructure systems in responding to events that disrupt services provided by the systems they manage. The decision makers in EROs are responsible for developing strategies for response and restoration and proposing them for review by stakeholders or regulators both within and external to their organization [5,6]. Once a strategy has been determined, it is implemented by field personnel. The computer-based decision aid being developed in this research maintains the independent system perspective for managers of each system, while providing the interdependent view for persons charged with setting priorities and directing restoration activities when an event impacts two or more of these systems simultaneously, e.g. the New York City Office of Emergency Management.

Officials at Verizon and Consolidated Edison were interviewed as part of the model development and provided data and insights for the case used to assess the model's contruct. It is anticipated that these same officials as well as representatives of the New York State Emergency Management Office and New York City Office of Emergency Management will be used to evaluate the models as part of our activities in the coming years.

Interdependent infrastructures can be viewed as networks, with movement of commodities (i.e. material) corresponding to flows and with services corresponding to a desired level of these flows. For each commodity, each node is either a supply node which is a source for the commodity; a demand node which is a point that requires some amount of the commodity; or a transshipment node which is a point that neither produces nor requires the commodity but serve as a point though which the commodity passes [7]. Arcs may, of course, have limited capacities. Infrastructure systems operate in an environment subject to disruptions, natural, human-caused or willful acts. Based upon performance criteria, an infrastructure system can be designed to minimize possible service degradation following a disruption. In addition, once a disruption occurs, alternative ways of restoring service can be determined.

Three models are presented. Section 2 details the normal operations model, prior to any disruption. A model for responding to a disruption is presented in section 3, and this model is intended to aid EROs in rapidly prioritizing among the possible responses. The response model does not directly consider interactions between different infrastructures. The restoration model of section 4 is for actions taken in response to the disruption that require consideration of interactions among and prioritizing across the different services provided by the infrastructures. It is in this section that five different kinds of interdependencies are described, and the model includes these interdependencies. Section 5 contains an example of the models. The paper concludes with suggestions for future research.

2. Normal Operations Model

Mathematically, a collection of infrastructure systems is represented as follows. Let I denote the set of infrastructures. Infrastructure $i \in I$ has nodes V^i and directed arcs E^i. Associated with each node $j \in V^i$ is a scalar b^i_j representing its supply or demand. If node $j \in V^i$ is a demand point then $b^i_j < 0$; if it is a supply point then $b^i_j > 0$; and if it is a transshipment node then $b^i_j = 0$. If $j \in V^i$ is a supply node then b^i_j equals the maximum possible amount that could be produced at that node. A nonnegative vector of variables, x^i_e, represents the flow on each arc e of the infrastructure. Associated with each arc e in E^i are non-negative scalars of costs c^i_e and capacities u^i_e, where $0 \leq x^i_e \leq u^i_e$.

Arcs are represented using either the endpoints of the arc or the index of the arc. For a node $l \in V^i$ for some infrastructure $i \in I$, let $\delta^+(l)$ denote the set of arcs in E^i that enter node l and let $\delta^-(l)$ denote the set of arcs in E^i that leave node l. Define $\delta(l) := \delta^+(l) \cup \delta^-(l)$, the set of all arcs incident to node *l*. Without loss of generality, assume that every supply node has no incoming arcs (i.e., $\delta^+(l) = 0$ if $b_l^i > 0$) and that demand nodes have no outgoing arcs, (i.e., $\delta^-(l) = 0$ if $b_l^i < 0$). A transshipment node *j* may have a limited capacity, w_j^i, modeled by placing an upper bound on total flow across the arcs $\delta^+(l)$. Included in the model are *flow conservation constraints* (i) that for supply nodes ensure that total flow out of the node is no greater than the available supply, (ii) that for demand nodes ensure that demand is met, and (iii) that for transshipment nodes ensure that flow into the node equals flow out of the node. The structural requirements are modeled by constraints on the capacities of arcs and transshipment nodes.

The objective during normal operations of a civil infrastructure system is to find the minimum cost feasible network flow. The complete representation of minimum cost network flow for each infrastructure $i \in I$, where the total flow into node *j* is given by $\sum_{e \in \delta^+(j)} x_e^i$ and the total flow out of the node is given by $\sum_{e \in \delta^-(j)} x_e^i$, is as follows:

$$\text{minimize} \quad \sum_{e \in E^i} c_e^i x_e^i \tag{1}$$

$$\text{subject to} \quad \sum_{e \in \delta^-(j)} x_e^i \le b_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i > 0 \tag{2}$$

$$\sum_{e \in \delta^+(j)} x_e^i = -b_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i < 0 \tag{3}$$

$$\sum_{e \in \delta^+(j)} x_e^i - \sum_{e \in \delta^-(j)} x_e^i = 0 \qquad \text{for } j \in V^i \text{ with } b_j^i = 0 \tag{4}$$

$$\sum_{e \in \delta^+(j)} x_e^i \le w_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i = 0 \tag{5}$$

$$x_e^i \le u_e^i \qquad \text{for } e \in E^i \tag{6}$$

$$x_e^i \geq 0 \qquad \text{for } e \in E^i \qquad (7)$$

Under normal conditions, all demands of all infrastructures are met. Referring back to the definition of input dependency, if all demands are met then all the interdependent components operate. So when all demands are met, the systems can be looked at as operating independently. It is only when failures occur that interdependencies become a concern. This *normal operations model* provides the baseline representation of an infrastructure.

3. Response to a Disruption

To be of use in addressing disruptions, the normal operations model is reformulated by the addition of slack variables and the capability to weight these variables so that emergency managers in the Emergency Response Organization (ERO) may prioritize. These weighting factors cause the model to attempt to reduce a priority demand's slack to zero first, before meeting demands with lower priority. Therefore the *response model* is given by: for infrastructure $i \in I$, where the total flow into node *j* is given by $\sum_{e \in \delta^+(j)} x_e^i$ and the total flow out of the node is given by $\sum_{e \in \delta^-(j)} x_e^i$,with s_j^i , as slack variables and weighting factors k_j^i, the *response model* is as follows:

$$\text{minimize} \qquad \sum_{e \in E^i} c_e^i x_e^i + \sum_{j \in V^i} k_j^i s_j^i \qquad (8)$$

$$\text{subject to} \qquad \sum_{e \in \delta^-(j)} x_e^i \leq b_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i > 0 \qquad (9)$$

$$s_j^i + \sum_{e \in \delta^+(j)} x_e^i = -b_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i < 0 \qquad (10)$$

$$\sum_{e \in \delta^+(j)} x_e^i - \sum_{e \in \delta^-(j)} x_e^i = 0 \qquad \text{for } j \in V^i \text{ with } b_j^i = 0 \qquad (11)$$

$$\sum_{e\in\delta^{+}(j)} x_e^i \le w_j^i \qquad \text{for } j\in V^i \text{ with } b_j^i = 0 \qquad (12)$$

$$x_e^i \le u_e^i \qquad \forall i\in I \text{ and } \forall e\in E^i \qquad (13)$$

$$x_e^i \ge 0 \qquad \forall i\in I \text{ and } \forall e\in E^i \qquad (14)$$

$$s_j^i \ge 0 \qquad \forall i\in I \text{ and } \forall j\in V^i \qquad (15)$$

where the symbolic representations are the same as the those for the normal operations model.

4. Interdependencies and the Restoration Model

If all revised demands for all infrastructure services can be met, each infrastructure system is considered to be operating independently. However, when unmet demand for any infrastructure service is found, interdependencies among infrastructure systems are considered and incorporated in order to support the restoration decision-making process.

1) Input: In general, input dependency is represented as follows: Define the set $V^{i,+} \subseteq V^i$ to be the nodes $j\in V^i$ with $b_j^i > 0$ (supply nodes). Sets $V^{i,=} \subseteq V^i$ (transshipment nodes) and $V^{i,-} \subseteq V^i$ (demand nodes) are defined similarly. Let $D(i,i_1) \subseteq V^{i,-}$ be the set of nodes in i that some other infrastructure i_1 depend upon (parent nodes) and let $D^i := \cup_{i_1\in I, i_1\ne i} D(i,i_1)$. This subset of nodes is the interdependent nodes. The remaining nodes in $V^{i,-}$ will be referred to as the independent nodes. The binary variable $y_{i_1,j}^{i,l}$ is the connection between node l in infrastructure i (where it is a demand node) and node j in infrastructure i_1, where it may be either a supply, demand or transshipment node and is only defined for $l \in D(i, i_1)$.

Let $C(i_1, i) \subseteq V^{i_1}$ be the set of nodes in i_1 that depend on some other infrastructure i, (child nodes) and let $C^{i_1} := \cup_{i \in I, i \neq i_1} C(i_1, i)$.Without loss of generality, all nodes have been disaggregated to the point where, given infrastructures i, i_1, and l in $D(i,i_1)$, there is a unique node j in $C(i_1,i)$ such that $y_{i_1,j}^{i,l}$ is defined, and given infrastructures i, i_1, and node j in $C(i_1,i)$, there is a unique node l in $D(i,i_1)$, such that $y_{i_1,j}^{i,l}$ is defined. Let $F(i, i_1)$ be the set of ordered pairs (l,j) associated with node l in $D(i,i_1)$ and node j in $C(i_1,i)$ for each $y_{i_1,j}^{i,l}$.

The objective function of the restoration model incorporates different priorities in addition to modeling interdependencies. On independent nodes, the available supply may be meeting the required demand or there may be some shortfall. The slack variable s_j^i represents the shortfall in meeting demands at independent nodes. In the model, there is no consideration for partial slack at the interdependent nodes. Because these interdependent nodes control the operation of nodes in other infrastructure systems, if they are not fully operational then they are in a failed condition: there is no benefit to partially meeting the requirement. Following the response phase, when there are unmet demands across one or more systems, one choice for the objective function is to minimize the total shortfall (slack) plus the unmet interdependent demands. A *restoration model* is defined as follows:

$$\text{minimize} \qquad \sum_{i\in I}\sum_{j\in V^{i,-}\setminus D^i} k_j^i s_j^i + \sum_{i\in I}\sum_{l\in D^i}\sum_{i_1\neq i} b_l^i (1-y_{i_1,j}^{i,l}) \tag{16}$$

subject to

$$\sum\nolimits_{e\in\delta^-(j)} x_e^i \le b_j^i \qquad \forall j\in V^{i,+}, \forall i\in I \tag{17}$$

$$\sum_{e\in\delta^+(j)} x_e^i - \sum_{e\in\delta^-(j)} x_e^i = b_j^i \qquad \forall j\in V^{i,=}, \forall i\in I \tag{18}$$

$$s_j^i + \sum_{e\in\delta^+(j)} x_e^i = -b_j^i \qquad \forall j\in V^{i,-}, \forall i\in I \tag{19}$$

$$\sum\nolimits_{e\in\delta^+(j)} x_e^i \le w_j^i \qquad \forall j\in V^{i,=}, \forall i\in I \tag{20}$$

$$\sum\nolimits_{e\in\delta^-(j)} x_e^{i_1} \le b_j^{i_1} y_{i_1,j}^{i,l} \qquad \forall (l,j)\in F(i,i_1) \text{ with } b_j^{i_1} > 0, \forall i, i_1\in I, i\neq i_1 \tag{21}$$

$$s_j^{i_1} + \sum_{e\in\delta^+(j)} x_e^{i_1} = -b_j^{i_1} y_{i_1,j}^{i,l} \qquad \forall (l,j)\in F(i,i_1) \text{ with } b_j^{i_1} < 0, \forall i, i_1\in I, i\neq i_1 \tag{22}$$

$$\sum_{e\in\delta^+(j)} x_e^{i_1} \le w_j^{i_1} y_{i_1,j}^{i,l} \qquad \forall (l,j)\in F(i,i_1) \text{ with } b_j^{i_1} = 0, \forall i, i_1\in I, i\neq i_1 \tag{23}$$

$$s_l^i \le (1-y_{i_1,j}^{i,l}) b_l^i \qquad \forall (l,j)\in F(i,i_1), \forall i, i_1\in I, i\neq i_1 \tag{24}$$

$$x_e^i \le u_e^i \qquad \forall e\in E^i, \forall i\in I \tag{25}$$

$$x^i \ge 0 \qquad \forall i\in I \tag{26}$$

$$y_{i_1,j}^{i,l} \qquad \text{binary, } \forall (l,j)\in F(i_1,i), \forall i, i_1\in I, i\neq i_1 \tag{27}$$

$$s_j^i \ge 0 \qquad \forall j\in V^i \text{ with } b_j^i < 0, \forall i\in I \tag{28}$$

For the remaining four interdependencies, their mathematical representations are as follows.

2) Mutual Dependence: A collection of infrastructures is said to be mutually dependent if at least one of the activities of one infrastructure system is dependent upon any other infrastructure system and at least one of the activities of this other infrastructure system is dependent upon the first infrastructure system. So in the case of two systems i and i_1 , mutual dependence would occur if there is at least one $y^{i,l}_{i_1,j}$ (connection between node l in infrastructure i (where it is a demand node) and node j in infrastructure i_1) and at least one $y^{i_1,m}_{i,n}$ (connection between node m in infrastructure i_1 (where it is a demand node) and node n in infrastructure i). Consider a natural gas system pump and a gas-fired electric power generator. From the perspective of the natural gas system, the pump is a transshipment node and the generator is a demand node. From the perspective of the electrical network, the generator is a supply node and the pump is a demand node. The generator needs gas to produce electricity; the pump needs electric power to deliver gas through the system to the generator. In this case $y^{i,l}_{i_1,j}$ would be the connection from the power system (infrastructure i) to the pump in the gas system (infrastructure i_1), and $y^{i_1,m}_{i,n}$, the connection from the gas system (infrastructure i_1) to the generator in the power system (infrastructure i). Failure of one component causes its corresponding binary variable to be set to zero, thus reducing the effective capacity of the other component to zero. In other words, if the pump were to fail, supply of gas to the generator would be inadequate and $y^{i_1,m}_{i,n}$ would be set to zero. When $y^{i_1,m}_{i,n} = 0$, the capacity of the generator is set to zero (since its effective capacity is the product of $y^{i_1,m}_{i,n}$ and its rated capacity b). Because the generator is a supply node, all flows on the arcs (i.e., the power lines) leaving the generator would now be zero, by flow conservation. Alternately, a lack of power at the pump demand node in the electrical generating network causes its binary variable $y^{i,l}_{i_1,j}$ to be set to zero and the capacity of the pump to be set to zero. To correct this situation, either an alternate source of gas must be found for the generator or an alternate source of power must be found for the pump.

3) Co-located: The co-located interdependency occurs when any of the physical components or activities of the civil infrastructure systems are situated within a prescribed geographical region. It was previously noted that managers of individual infrastructure systems would identify the components of their respective system at or near the site of the incident which may have been affected by the event. Based on further investigation, the status of these components will be adjusted. However, since only those EROs who are responsible for coordinating activities across multiple agencies maintain the complete view of all civil infrastructure systems, it is ultimately their responsibility to ensure that all co-located interdependencies have been considered and the models of the affected infrastructures revised as appropriate.

4) Shared (AND): Shared interdependence occurs when some physical components and/or activities of the infrastructure used in providing the services are shared. In the context of the telecommunications and power systems and the WTC restoration, it could have been advantageous to route the shunts used to restore phone and power through the same temporary enclosures. This might have reduced time and cost since only one enclosure would need to be built, but could then lead to coordination problems between the two companies. This situation could be modeled by changes in the objective function and constraint equations.

5) Exclusive-or (XOR): When multiple services share infrastructure component(s), for example, an arc, but the component can only be used by one service at a time, exclusive-or interdependence occurs. Considering power and telecommunications, it can also be the case that a power and a telecommunications shunt may not be able to be routed in close proximity to each other. This would be the case with a T-1 line and a high voltage distribution line which can not be too close together due to RF interference considerations. Exclusive-or interdependencies are modeled by selecting additional constraints to restrict flow to one commodity or the other.

5. An Illustrative Example

This section presents an example drawn from cases of infrastructure interdependency that arose following the World Trade Center attack, as reported by *The New York Times*. Additional information was obtained from interviews with Consolidated Edison (ConEd) and Verizon personnel. Much of the data associated with the attack (e.g., locations of equipment and personnel, generating capabilities, capacities of feeder lines and shunts, power demands) is sensitive and has not been used. In order to illustrate modeling of each of the five interdependencies, a simulated event is used, a monitoring system (telecommunications) failure leading to losses of power. The following case illustrates how the response and restoration models could be used to provide decision support to infrastructure operators and emergency managers. This case is described in greater detail in [8].

The first system used in this case is the power distribution system. This system has four high voltage power supplies. This high voltage is the input to multiple substations, which transform down the high voltage power received from the transmission system to 13,500 volts (13.5 kV). From these substations, power is provided to 120/208 volt transformers, and then to the customers. The customers include the New York Stock Exchange (NYSE), a hospital, One Police Plaza (the New York City Police Headquarters), two facilities of the Metropolitan Transportation Authority (transit services), six Controlled Environmental Vaults (CEV) of the phone company (described below), and two general residential areas and two general business areas.

The second system is the telephone system, in which customers function as both supply and demand nodes. The system has 18 neighborhood areas consisting of both residential and business customers. Calls originate at a customer and are collected along a distribution cable typically serving dozens of customers. Many distribution cables come together at a Controlled Environmental Vault (CEV). Calls then pass through a feeder cable containing thousands of lines and come together in the cable vault of a central office and into a switching system. From the central office, they pass to one of the following: to another central office through an interface trunk; to a tandem[3] via a trunk link; or out through the same set of CEVs that feed the (originating) central office.

To illustrate a mutual dependence of telecommunications on power and power on telecommunications, assume a failure in the power distribution system which causes the

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[3] A tandem has trunk lines to all central offices in its sector and trunks to all other tandems with the same of other companies providing service to the world network.

failure of a Controlled Environment Vault (CEV) in the phone system. The failure of the CEV results in a loss of telephone service. With this failure in the phone system, the supervisory control and data acquisition (SCADA) system for the power company becomes unreliable, causing loss of reliable indicators on a set of distribution transformers and causing breakers to malfunction. (This failure was not observed during the WTC attack and is only inserted to illustrate mutual dependency.) The disruption causes the failure of one substation, the power supply line to two CEVs (denoted D and E) , and the phone feeder lines from CEV D and two other CEVs (denoted B and C) to a Central Office.

As part of the impact assessment, field observers report that the substation is completely destroyed. They also report that they are unable to ascertain the condition of the four lines that were affected due to the extensive debris but are confident that the lines are not serviceable. The system operators at Verizon and ConEd make modifications to the response phase model, using data from the field. The supply at this substation and the capacity of the one electric and three phone lines are reduced to zero. Each manager runs the response model on his or her system independent of the other.

The results show unmet demand at the New York Stock Exchange, residential areas, a hospital, and CEVs C, D, and E based on the slack variables in the power system model. In the phone system, the operator notes there is unmet demand in the neighborhoods served by CEVs B, C, and D. In the response model discussed earlier, it is noted that prioritization can be done in an attempt to meet vital loads at the expense of less important loads, if appropriate.

With unmet demands in their respective systems and no feasible solutions, the operators provide these data to the emergency response organizations that then move the event to the next stage – restoration. The operator uses the restoration model to identify the slack variables corresponding to unmet demands in the two systems, including interdependencies. Since the effects on these two systems are being considered, the input dependencies are the loss of power to CEVs C and D. CEV E contains the SCADA system for a second substation. Following the loss of power to CEV B, the loss of SCADA causes a loss of reliable indication and control of the output breakers causing them to open resulting in loss of power to all components served by this second substation.

When the emergency response organization runs the restoration model, the results indicate the unmet demands noted by the infrastructure managers as well as several new, unmet demands. The loss of power to CEV C via its input dependence results in loss of service in the neighborhoods it serves. Failure of CEV E and the SCADA system leads to new failures in the power grid and unmet demand at Metropolitan Transportation Authority (MTA) trains and stations (subway services) and One Police Plaza (police headquarters). NYCOEM personnel now move to the second portion of the restoration phase. Available resources are identified and restoration strategies are developed, in consultation with the individual system infrastructure managers. These strategies consist of new lines and temporary power sources.

Priority for the power company is restoring power to the New York Stock Exchange, CEV C, CEV D, CEV E, and the hospital (restoring power to CEV E will restore SCADA to the second substation and therefore, restore power to One Police Plaza and the MTA facility). The power company has also received a request to provide new power lines to the area of the disruption for rescue operations (lighting, pumps, etc). The power company determines that three substations can be used to provide some of this power: the second substation is available to provide 125 units of power beyond its current loads (once SCADA is restored and power is provided to One Police Plaza and MTA); for the two additional substations, one can provide 50 units of power and the other can provide 60 units beyond their current loads. CEVs C, D and E each require 10 units of power; the New York Stock Exchange requires 100; the hospital requires 125 and the World Trade Center site needs 50.

The phone company is focused on restoring the lines between CEVs B, C, and D and the Central Office. All of the temporary lines run must be housed in enclosures to ensure the safety of the public and to protect the lines from damage. Phone shunts which contain only voice circuits, known as POTS, can be housed in the same enclosure as the power line. However, any phone line containing a T-1 line must not be run in the vicinity of a power line, due to interference. These requirements serve as the bases for the remaining two interdependencies, shared (AND) and exclusive-or (XOR). The POTS lines and power are modeled as a shared interdependency; that is, the lines will be run together when possible. The T-1 line and power line are an exclusive-or interdependency, where only one line or the other may be routed along a particular path.

The power company has sufficient shunts to connect from the three available substations to each of the loads. Each shunt from a supply to a demand node has its associated cost and capacity. There are also two diesel generators available and four suitable sites for them. Each site could have one or both generators, allowing for two one-generator sites or one two-generator site.

The phone company has sufficient resources to reconnect the three CEVs to the central office. Due to the location of the failure, the company also has an option to connect to another central office versus the original connection. There are multiple routing choices available for each connection, each having its own associated cost. However, because this portion of the restoration is being done in conjunction with the power company, both the XOR constraint of only one system's line being located along some paths and the AND constraint of both systems' lines being in the same enclosure must each be taken into account.

Based on discussions with domain experts, reasonable cost estimates were determined. The decision situation facing the emergency managers is, in essence, to construct a new network utilizing the working sections of the infrastructures and supplementing them with new shunts and temporary diesel generators. The specific objective function for this example is to minimize cost of operation of the shunts and the generators.

Utilizing the modeling language AMPL and the CPLEX solver [9], the model determines the following course of action: From the second substation, connect a shunt to the New York Stock Exchange (NYSE). From each of the two additional substations, connect two shunts to various demand nodes. Place two diesels at one of the sites and supply the remaining needs of two of the demand nodes. The phone company runs its shunts from CEV B to the original central office and from CEV C and D to the alternate office. This plan meets the needed demands and does not violate the exclusive or requirements.

The model can also provide decision makers with alternatives. The effects of changes in the current situation can be evaluated. This is usually referred to as sensitivity analysis. For example, if the loads increase, the current solution could become infeasible (it will no longer meet the requirements). Reducing loads could lead to new, cheaper solutions. How much of a change is required to affect the current solution is determined by the sensitivity analysis. Managers could also use sensitivity analysis and new constraints to handle other contingencies such as more generators becoming available or shunts above some length (cost) should not be installed.

6. Summary

Models can provide powerful means of understanding [10], monitoring and controlling large-scale infrastructure systems [11]. The need for powerful but parsimonious models is particularly acute as modeled infrastructures increase in complexity, as when a number of infrastructures are interdependent. The particular focus of this work is on developing techniques that can be used to respond to and restore from events that have the capability of impacting interdependent infrastructure systems. The approach taken is to model salient elements of interdependent critical infrastructure systems and to provide decision makers with means of manipulating this model for purposes of response and restoration of service, i.e. a decision support system. The restoration model is an integer programming problem, which can be attacked using branch-and-cut [12].

These models are designed to be imbedded in a decision support system that will employ a database management system for storing data and information on response and restoration resources and have as the human-machine interface, a geographical information system. Emergency managers and infrastructure operators will then be able to see the full impact of actions across multiple systems and work collaboratively to provide the solutions that are best for all. Additionally, it is envisioned that this decision support system will have the capability of aiding system designers in increasing the resilience of their systems and increasing their awareness of the effect interdependency plays in the design and operation of these complex systems. Further algorithmic and model development will include consideration of time-expanded networks [7] and techniques used in network design and multicommodity network flows such as column generation and branch-and-price [13,14,15].

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