

An Interdependent Layered Network Model for a Resilient Supply Chain

Jing Gong^{1,2}, John E. Mitchell³, Ananth Krishnamurthy⁴, and William A. Wallace⁵

¹School of Public Administration & Emergency Management, Jinan University, China

²GuangDong Emergency Technology Research Center of Risk Evaluation and Prewarning on Public Network Safety, China

³Department of Mathematical Sciences, Rensselaer Polytechnic Institute, USA

⁴Department of Industrial and Systems Engineering, University of Wisconsin-Madison, USA

⁵Department of Industrial and Systems Engineering, Rensselaer Polytechnic Institute, USA

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Abstract

This paper addresses the problem of designing supply chains that are resilient to natural or human-induced extreme events. It focuses on the development of efficient restoration strategies that aid the supply chain in recovering from a disruption, thereby limiting the impact on its customers. The proposed restoration model takes into account possible disruptions to infrastructures, e.g., transportation and communications, by explicitly formulating their logical relationships with supply chains. A problem solving process is proposed that provides for cooperation between the managers of the infrastructures and the managers of the supply chains disrupted by an extreme event. Both the model and the problem solving process are exercised with a realistic industry problem.

Keywords: Supply Chain Management, Interdependent Layered Networks, Network Flows.

1 Introduction

This paper addresses the design of a resilient supply chain by proposing efficient restoration strategies to cope with supply chain disruptions. A supply chain comprises different entities that are connected by the physical flow of materials or products. Disruptions could occur at any section of the network, in any of the processes, for a wide variety of reasons such as transportation delays, power outages, or natural or man-made disasters. A resilient supply chain is a system that has the ability to recover quickly from disruptions and ensure customers are minimally affected.

Craighead and colleagues [1] studied the severity of the disruption and related it to three supply chain design characteristics: density, complexity, and node criticality; and to the two supply chain mitigation capabilities: recovery and warning. They pointed out that, as recovery capacity increases within a supply chain, the quicker the supply chain returns to the normal level and the less severe the disruption will likely be. They conclude that “*an unplanned event that disrupts a supply chain with the capability to respond quickly and effectively is less likely to be severe than the same supply chain disruption affecting a supply chain with little or no capability to recover*”. It can be illustrated by the respective responses of Nokia and Ericsson to the loss of a supply of radio-frequency chips (RFC) in early 2000 [2],[3]. Although facing the same situation, two companies responded differently and thus ended up with two endings: one survived from the disruption while the other ultimately exited from the business.

Infrastructure is defined as the set of interdependent networks and systems comprising identifiable industries, institutions (including people and procedures), and distribution capabilities that provide a reliable flow of products and services essential to the defense and economic security of a country, the smooth functioning of government at all levels, and society as a whole [4]. We approach the question of efficient restoration strategies raised by a resilient supply chain from the perspective of infrastructure systems and their logical relationship with the supply chain. Since the functioning of society depends heavily on energy, transportation, telecommunication, financial, and other infrastructures, infrastructure systems play an important role in operations of a supply chain. Ignoring these fundamental systems will make the study of supply chain management unrealistic and impractical, especially for supply chain restoration.

In the example of 3J's Trucking Company (3J's) [5], we can see that in order to restore its destroyed distribution system, its logistical scheduling system had to work effectively. However, the disruption to the telecommunication service caused 3J's to be unaware of road information and thus unable to devise alternate routes. Telecommunication restoration is critical to 3J's. It would bring back timely and accurate transportation

information, which leads to efficient logistical schedules. Ignoring telecommunication restoration, as well as its influence, would eventually cost the company both more money and time in the restoration of its distribution system. In the example of the Northeast Blackout of 2003 [6], the loss of the power supply caused the loss of production capacity of the factories in the affected area. Those factories couldn't restore their production until the power grid was stabilized. They were dependent on restoration of the power grid. If this dependency was taken into account—in other words, power restoration information was considered by supply chain managers during their restoration planning—they would have made better decisions concerning supply, inventory, and distribution. For example, they could have scheduled supply to arrive just as power in a region was restored and avoid paying for storing materials before manufacturing can be started, or lacking supply in the first few days of restoration of power.

Consideration of infrastructure systems in supply chain restoration will raise the following questions: How can we represent relationships between infrastructure systems and the supply chain? How can supply chain managers utilize information from infrastructure managers to make efficient restoration plans? And how can their plans benefit infrastructure managers? Our approach in addressing those questions seeks to achieve two goals: 1) develop a mathematical representation of logical dependencies between the supply chain and infrastructure systems, and 2) provide the best restoration strategy to help the supply chain managers and infrastructure managers cooperate to mitigate the impact of a disruption.

2 Literature Review

Supply chain design involves determining the supply chain configuration and the distribution of resources over the resulting supply chain network. Basically, it is a two-stage network design problem. First, managers decide on locations where facilities will be established and on the capacity to be assigned to each facility. Second, they assign cur-

rent demand to the available facilities and identify lanes along which products will be transported. How do they design a supply chain network to make maximum profits? Different companies gave their own answers. Pfizer, Inc. [7] modeled its distribution system as a two-echelon network which included its two large distribution centers, over 35 third-party pool distribution locations, and its customers' receiving locations. Kellogg Company [8] developed an operational planning system to help determine where products are produced and how finished products and in-process products are shipped between plants and distribution centers, which reduced production, inventory, and distribution costs by an estimated \$4.5 million in 1995. There are many companies using optimization techniques to design their supply chain, such as Deere's Commercial and Consumer Equipment (C&CE) Division's inventory management system [9], and Hewlett-Packard's supply chain [10]. Research on complicated issues involved in supply chain network design can be found in recent papers. For example, Pan and Nagi [11] considered a supply chain network design problem in an agile manufacturing scenario with multiple echelons and multiple periods under a situation where multiple customers have heavy demands. The problem is formulated to integrate decisions on selection of one or more companies in each echelon, production, inventory, and transportation, and minimize the total operational cost. Liu and Papageorgiou [12] addressed the issue of multiple objectives in the global supply chains, i.e., considering cost, responsiveness and customer service level simultaneously. A multi-objective mixed-integer linear programming (MILP) approach is developed with total cost, total flow time and total lost sales as key objectives, and the ε -constraint method and lexicographic minimax method are used as solution approaches to tackle the problem.

Recent literature points out the need for the design of a resilient supply chain [3],[2],[13]. Past research on supply chains that deals with disruptions has taken two perspectives. Some researchers viewed disruptions as one type of supply chain risk and discussed them from the perspective of risk management. Harland and colleagues [14] provided a review of risk in supply networks and discussed a theoretical tool that helps in the assessment

and management of the risk. Sinha and colleagues [15] presented a generic prescriptive methodology for mitigating disruption risk in an aerospace supply chain based on methodologies from risk management and supply chain management. Chopra and Sodhi [16] classified supply chain risks into different categories, identified drivers of these different risk categories, and discussed risk mitigation strategies. They defined ‘disruptions’ as disruptions to material flows anywhere in the supply chain and classified them into one risk category. Kleindorfer and Saad [17] developed a conceptual framework for risk assessment and risk mitigation. They identified three categories of supply chain strategies available to address disruption risk based on application areas. Restoration strategies proposed in this paper belong to the third category, the operational control of the supply chain, including emergency (or crisis) response.

Other researchers addressed disruption issues from the perspective of uncertainty of supply chain operations. Snyder and colleagues [18] gave an overview of designing supply chain networks against uncertainty of supply or demand and facility failures. The risk of uncertainty of disruptions is measured by two methods: expected cost and worst-case cost. The models identified optimal strategies for allocating limited resources among possible investments. Santoso and colleagues [19] proposed a stochastic programming approach for supply chain network design under uncertainty. They integrated a sampling strategy, the sample average approximation (SAA) scheme, with an accelerated Benders decomposition algorithm to quickly compute high quality solutions to large-scale stochastic supply chain design problems with a huge (potentially infinite) number of scenarios. In Moinzadeh and Aggarwal [20] disruptions refer to process unreliability, the effect of unreliability on production systems, including machine failure, labor strikes, and other breakdowns in production. The authors modeled those process disruptions as randomly distributed events and the production inventory system as a stochastic process. Sawik [21] addressed a resilient supply portfolio by selecting suppliers to be protected against disruptions and the allocation of emergency inventory of parts to be pre-positioned at the protected suppliers. Tomlin [22] developed a supply chain model

to investigate mitigation and contingency strategies based on a single-product setting with two supplier options: one is unreliable and another is reliable but more expensive. The author focused on supply-side tactics for managing disruption risks caused by supply uncertainty. Georgiadis and colleagues [23] modeled demand uncertainty in terms of a number of likely disruption scenarios and investigated their impact on the design of supply chain networks. Nickel and colleagues [24] considered uncertainty for demand and interest rates in the supply chain network design. The problem is formulated as a multi-stage stochastic mixed-integer linear programming problem, with the objective of maximizing the total financial benefit including return on investment and the service level. Goh and colleagues [25] developed a unified multi-stage stochastic model of a typical global supply chain network, taking into account the attendant uncertainties in market demand, volatility in exchange rates, differing country tax rates, and varying import tariffs at different ports of call even within a country.

Some papers discussed increasing flexibility to cope with supply chain disruptions. Stevenson and Spring [26] provided a more complete definition of flexibility in the context of supply chains and presented a general review of research on supply chain flexibility. Jordan and Graves [27] analytically compared different scenarios based on a planning model for assigning production to plants and developed several principles of manufacturing flexibility. Beach and colleagues [28] reviewed the theory and methodology of manufacturing flexibility and examined issues related to it, specifically, its relationship with environmental uncertainty. They developed a conceptual framework of manufacturing flexibility. Hallgren and Olhager [29] conducted an empirical analysis on relationships among volume and product mix flexibility, flexibility source factors, and operational performance. Iravani and colleagues [30] studied the structural flexibility that can be created by using multipurpose resources such as cross-trained labor, flexible machines, or flexible factories. They use the structure of the capability pattern to explore the ability of a system to respond to variability in its environment. Yu and colleagues [31] focused on the structural flexibility created by flexible retailers in the distribution network where one

retailer's excess inventory can be transferred to satisfy other retailers unmet customer demand due to stock-outs. They investigated under different levels of the flexibility how ordering policy affects the firm's performance. Snyder and colleagues [32] surveyed recent developments in supply chain disruption research.

This paper addresses the design of a resilient supply chain by developing efficient restoration strategies that aid the managers of a supply chain in recovering from a disruption, thereby limiting the impact on its customers. We argue that supply chain restoration is a function of the infrastructure systems. In addition, restoration needs to consider all phases of the supply chain operation, including production, inventory, transportation, and distribution. This research on supply chain management is unique in that it considers infrastructures and their influences on the supply chain. The supply chain network is modeled not in isolation, but is considered dependent on infrastructures, and these dependencies are explicitly represented. We propose a framework for generating efficient restoration strategies, which involve not only redesigning the network and decision making for production, inventory, and distribution, but also cooperation between the supply chain managers and management of infrastructures to mitigate the impact of a disruption.

3 Interdependent Layered Network Model

Generally, the graphical representation of a supply chain consists of a single-layer network. Nodes could be production facilities, warehouses, or demand zones. Arcs in the network represent connectivity between different nodes. Flows on the individual arcs represent the movement of materials or products between nodes connected by arcs. One limitation of the single-layer network structure is the inability to represent relationships between supply chains and other support infrastructure networks that influence supply chain operations, such as power, communication, and so on. To model these dependencies, a different structure, Interdependent Layered Networks (ILN), is employed in this

paper. The concept of ILN was first proposed by Lee and colleagues in the restoration of interdependent infrastructure systems [33].

ILN is composed of multiple networks, with each network identified as a layer. There exist logical relationships that connect layers. ILN is designed to highlight such logical relationships and capture their cascading impacts. Based on the conceptual model of ILN, we propose the following compact formulation for restoration of the supply chain, with its supporting infrastructure networks explicitly taken into account. The restoration model can be written compactly as:

$$\min \sum_{i \in I} (c^{iT} x^i + h^{iT} y^i + r^{iT} s^i) + g^T z \quad (1)$$

$$\text{s.t.} \quad A^i x^i + B^i y^i + C^i s^i = b^i \quad \forall i \in I \quad (2)$$

$$\sum_{i \in I} (H^i x^i + D^i y^i + R^i s^i) + Gz = d \quad (3)$$

$$x^i \in X^i \quad \forall i \in I \quad (4)$$

$$y^i \in Y^i \quad \forall i \in I \quad (5)$$

$$s^i \in S^i \quad \forall i \in I \quad (6)$$

$$z \in Z \quad (7)$$

I is the set of all network layers involved, including the supply chain network layer, represented by ‘ sc ’, and infrastructure layers, represented by i_1 if single-commodity networks and i_2 if multi-commodity networks. $A^i, B^i, C^i, H^i, D^i, R^i, G$ are matrices. They are determined by the topological structure of ILN network, including layout of all nodes and arcs in each network layer, and logical connections among different layers. $c^i, x^i, h^i, y^i, r^i, s^i, g, z, b^i, d$ are column vectors. The size of vectors are also determined by ILN network topology. The variables x^i represent flows of network i . The variables y^i represent restorative decisions for network i . They could be the repair of destroyed power lines, outsourcing of production, or installation of temporary devices. The variables s^i model disruptions in terms of unmet demands. The variables z are used to model the various interdependencies. In general, the set X^i and S^i represents nonnegativity

while the set Y^i and Z represent nonnegative integrality. The first set of constraints are traditional flow conservation within each network. The second set of constraints link the various networks together, and represent interdependencies among them. Notice that since networks could be single-commodity, or multi-commodity, indexes of the same variable might be different in different layers.

The choice of objective function depends on who is solving the problem. An emergency manager looking to restore all the infrastructure systems would have a different objective from a supply chain manager. A supply chain manager would also not be able to choose values for all the variables: for example, we envision a situation in which an emergency manager determines values for the restorative decision variables y^i , where i represents the disrupted infrastructures, and then the values of these variables are presented to the supply chain manager as *parameters*. The supply chain manager's problem is presented in Section 4.1, and that of the infrastructure managers in Section 4.2. Their interaction is discussed in Section 4, with computational results in Section 5.

In the normal situation, networks are not affected by operations of other networks with the assumption that no disruptions occurs, so each layer of network can be treated independently. However, if events occur, causing disruptions, interdependencies among networks would quickly spread the impacts to other layers. From the perspective of service restoration, unmet demands could be the direct impact of events, or the cascading impact of interdependencies. Therefore, optimizing the restoration of any individual layer involves not only its own operations, but operations of its associated layers. In other words, a restoration model should consider all networks simultaneously due to the existence of unmet demands that manifest interdependencies and explicitly link all networks together.

The subsequent subsections discuss the application of this formulation in supply chain restoration, with consideration of three typical supporting infrastructures: power, telecommunication, and transportation. The reasons why these three infrastructures are chosen are twofold. One is they are critical to emergency restoration, with each of

them corresponding an ESF (Emergency Support Function) out of 15 ESFs defined in U.S.A. National Response Plan (2008). For example, we find, in testimony of U.S.A. Federal Emergency Management Agency administrator Craig Fugate at a House Committee hearing titled “A Review of the Preparedness, Response To and Recovery From Hurricane Sandy”, a majority of the infrastructure section focuses on these three infrastructures. The other reason is they are critical to the operations of supply chains, as illustrated in the following sections.

3.1 Supply chain network layer

The supply chain is modeled as a graph G^{sc} , where $G^{sc} = G(V^{sc}, E^{sc})$. The set of nodes V^{sc} includes plants, V_P^{sc} , warehouses, V_W^{sc} , and customer demand zones, V_D^{sc} . Arcs in E^{sc} denote connectivities between different locations. The manufactured products could be stored either at the plant or a warehouse, or directly sent to the demand zones to satisfy customer requirements. Basically, the supply chain model is an arc-based, multi-commodity, network flow layer. It is assumed that shipping lead times are much longer than the manufacturing lead time, so the time gap between releasing a production order and its availability at a demand zone is dominated by the shipping lead times.

Let T denote the whole time period in the planning horizon and $d_{j,k,t}$ denote the demand for product k in demand zone j during a particular time period t . The variables include $pro_{j,k,t}$ and $inv_{j,k,t}$ which denote respectively the production and inventory at node j for commodity k during time period t , and $x_{(j,l),k,t}^{sc}$ which denotes movement of product k on arc (j, l) during time period t . Please see Appendix A for a complete list of notation. The following are the constraints associated with the operations of the supply chain network layer:

For each product produced at a plant, the sum of production during a period and the beginning inventory must equal the sum of the inventory at the end of this period and

the total units shipped to warehouses and demand zones during this period:

$$pro_{p,k,t} + inv_{p,k,t-1} = \sum_{l \in V_W^{sc} \cup V_D^{sc}} x_{(p,l),k,t}^{sc} + inv_{p,k,t}, \quad \forall p \in V_P^{sc}, \forall k \in \phi(p) \text{ and } t \in T \quad (8)$$

where $\phi(p)$ denotes the set of products produced at plant p . For each product at a warehouse, the sum of beginning inventory and the units received from the plants during a period must equal the sum of the inventory at the end of this period and the total units shipped out during this period:

$$\sum_{p \in V_P^{sc}} x_{(p,w),k,t-tl(p,w)}^{sc} + inv_{w,k,t-1} = \sum_{l \in V_D^{sc}} x_{(w,l),k,t}^{sc} + inv_{w,k,t}, \quad \forall w \in V_W^{sc}, k \in K \text{ and } t \in T \quad (9)$$

At each customer zone, products available at a warehouse (or a plant) during a time period are shipped to meet that period's demand. If the demand cannot be met from available inventory, the unmet demand for product k at demand zone l at time t is considered as lost demand and is modeled by a non-negative slack variable $s_{l,k,t}$, which is assigned a penalty in the objective function.

$$\sum_{w \in \delta(l)} x_{(w,l),k,t-tl(w,l)}^{sc} + \sum_{p \in \delta(l)} x_{(p,l),k,t-tl(p,l)}^{sc} + s_{l,k,t} = d_{l,k,t}, \quad \forall l \in V_D^{sc}, \forall k \in K \text{ and } t \in T \quad (10)$$

We also have flow capacity constraints:

$$\sum_{k \in K} x_{(j,l),k,t}^{sc} \leq v_{(j,l)}, \quad \forall (j,l) \in E^{sc} \text{ and } t \in T, \quad (11)$$

production capacity constraints:

$$\sum_{k \in \phi(p)} pro_{p,k,t} \leq w_p, \quad \forall p \in V_P^{sc} \text{ and } t \in T, \quad (12)$$

production outsourcing constraints, where $V_{P'}^{sc}$ represent all potential outside manufacturers:

$$\sum_{k \in \phi(p)} pro_{p,k,t} \leq w_p * y_{p,t}^{sc}, \quad \forall p \in V_{P'}^{sc} \text{ and } t \in T, \quad (13)$$

and inventory capacity or storage space constraints:

$$\sum_{k \in K} inv_{l,k,t} \leq q_l, \quad \forall l \in V_P^{sc} \cup V_W^{sc} \text{ and } t \in T. \quad (14)$$

The products shipped out from a plant must be produced at this plant. The set $K \setminus \phi(p)$ represents products that are not produced at plant p :

$$\sum_{l \in V_W^{sc} \cup V_D^{sc}} x_{(p,l),k,t}^{sc} = 0, \quad \forall p \in V_P^{sc}, k \in K \setminus \phi(p), \text{ and } t \in T. \quad (15)$$

3.2 Power system network layer

A modern electric power system [33] consists of six main components: 1) the power station, 2) a set of transformers to raise the generated power to the high voltages used on the transmission lines, 3) transmission lines, 4) substations at which the power is stepped down to the voltage on the distribution lines, 5) the distribution lines, and 6) the transformers that lower the distribution voltage to the level used by the consumer's equipment. In this paper, the layer of power system, modeled as a single commodity network $G^{i_1} = G(V^{i_1}, E^{i_1})$, consists of power stations, transformers, substations, and transmission lines, not including low voltage distribution. Power stations are modeled as supply nodes, $V_S^{i_1}$, transformers as transshipment nodes, $V_T^{i_1}$, and substations as demand nodes, $V_D^{i_1}$. Power is assumed to be delivered through transmission lines to substations and each substation serves an area. Arcs, E^{i_1} , represent transmission lines connecting the different locations. Those in the event area that need to be repaired or replaced are represented by $E^{i_1'}$.

Equations (16)–(21) are single commodity flow conservation and capacity constraints. Equation (16) is the power node capacity constraint. Equation (17) is the power demand constraint with consideration of unmet demand, $s_{j,t}^{i_1}$. Binary variable $z_{j,t}^{i_1, i_1}$ indicates if an unmet demand exists and will be used later in modeling interdependencies. Equation (18) sets an upper bound of unmet demand $s_{j,t}^{i_1}$. Equation (19) sets the flow balance for the transshipment nodes. Equation (20) ensures that a damaged transmission line cannot

work until it is repaired. Binary parameter $y_{(j,l),t}^{i_1}$, $(j,l) \in E^{i_1'}$ indicates the decision on repairing a transmission line. The set of y^{i_1} reflects a restoration plan for the power system i_1 . Equation (21) is the arc capacity constraint.

$$\sum_{(j,l) \in E^{i_1}} x_{(j,l),t}^{i_1} \leq b_j, \quad \forall j \in V_S^{i_1} \text{ and } t \in T \quad (16)$$

$$\sum_{(l,j) \in E^{pr}} x_{(l,j),t}^{i_1} + s_{j,t}^{i_1} = m_j^{i_1}, \quad \forall j \in V_D^{i_1} \text{ and } t \in T \quad (17)$$

$$s_{j,t}^{i_1} \leq m_j^{i_1} * z_{j,t}^{i,i_1}, \quad \forall j \in V_D^{i_1}, i \in \varphi(i_1), t \in T \quad (18)$$

$$\sum_{(i,j) \in E^{i_1}} x_{(i,j),t}^{i_1} - \sum_{(j,l) \in E^{i_1}} x_{(j,l),t}^{i_1} = 0, \quad \forall j \in V_T^{i_1} \text{ and } t \in T \quad (19)$$

$$x_{(j,l),t}^{i_1} \leq v_{(j,l)} * y_{(j,l),t}^{i_1}, \quad \forall (j,l) \in E^{i_1'} \text{ and } t \in T \quad (20)$$

$$x_{(j,l),t}^{i_1} \leq v_{(j,l)}, \quad \forall (j,l) \in E^{i_1} \setminus E^{i_1'} \text{ and } t \in T \quad (21)$$

3.3 Telecommunication system layer

The communication network layer is modeled as a multi-commodity network G^{i_2} , where $G^{i_2} = G(V^{i_2}, E^{i_2})$. A typical communication system network consists of fiber spans [36]. A span is a fundamental physical component in the network connecting two terminals. A span may fail because of various causes, including cable cuts and natural disasters. Data traffic is routed on the network through a sequence of spans. In this paper, spans are modeled as arcs and terminals as nodes. A phone call is specified by an origin node and a destination node. For each pair of terminals (nodes), referred to as an OD (Origin and Destination) pair, several routes (or paths) may exist to deliver telecommunication service. Those with the same OD pair are modeled as one commodity. Different commodities mean phone calls have different origins and/or different destinations.

Equations (22)–(27) are basic multi-commodity flow conservation and capacity constraints with consideration of unmet communication demand, $s_{o,t}^{i_2}$, $o \in OD^{i_2}$. We use a

path-based formulation. Variable $f_{o,n,t}^{i_2}$ denotes the flow at time t on the n th path for the OD pair $o \in OD^{i_2}$. Equation (22) links path flows to arc flows. The flow on each arc is calculated by summing all flows along paths that contain this arc. Equation (23) defines the unmet demand for each OD pair. Equation (24) sets the upper bound of the unmet demand. Equation (25) is the node capacity constraint. Equation (26) ensures that a damaged span cannot work until it is repaired. Equation (27) is the flow capacity constraint. Binary variable $z_{o,t}^{i_2}$ indicates if there are unmet communication demands. Parameter $y_{(j,l),t}^{i_2}$, $(j,l) \in E^{i_2'}$ indicates the decision on repairing a span. The set of y^{i_2} reflects a restoration plan for the telecommunication network.

$$\sum_{n \in 1..N} \sum_{(o,n) \text{ contains } (j,l)} f_{o,n,t}^{i_2} = x_{(j,l),t}^{i_2}, \quad \forall (j,l) \in E^{i_2} \text{ and } t \in T \quad (22)$$

$$\sum_{n \in 1..N} f_{o,n,t}^{i_2} + s_{o,t}^{i_2} = m_o^{i_2}, \quad \forall o \in OD^{i_2} \text{ and } t \in T \quad (23)$$

$$s_{o,t}^{i_2} \leq m_o^{i_2} * z_{o,t}^{i_2}, \quad \forall o \in OD^{i_2}, i \in \varphi(i_2) \text{ and } t \in T \quad (24)$$

$$\sum_{(l,j) \in E^{i_2}} x_{(l,j),t}^{i_2} \leq b_j, \quad \forall j \in V^{i_2} \text{ and } t \in T \quad (25)$$

$$x_{(j,l),t}^{i_2} \leq v_{(j,l)} * y_{(j,l),t}^{i_2}, \quad \forall (j,l) \in E^{i_2'} \text{ and } t \in T \quad (26)$$

$$x_{(j,l),t}^{i_2} \leq v_{(j,l)}, \quad \forall (j,l) \in E^{i_2} \setminus E^{i_2'} \text{ and } t \in T \quad (27)$$

3.4 Transportation system layer

The nodes in the supply chain network correspond to the nodes in the transportation network, and the arcs in the supply chain network represent transportation connectivity. An arc in the supply chain network denotes a path connecting two locations. In practice, this path could be the shortest path, the least-time path, or any other suitably defined path. Therefore, the transportation system can be viewed as part of the supply chain.

When a disruption event occurs, connectivity in the transportation network could be

affected and thus decision makers have to consider alternate paths to move products. This necessitates an update of the supply chain network with new parameters, such as transshipment cost, capacity limit, and even the network structure if, for example, due to transportation disruption there is no way to move products between two locations. As a result, the arc between two corresponding nodes in the network will be eliminated. Decision makers will require flow distribution to be re-optimized through the updated network.

3.5 Modeling interdependencies

Let $DEP(i', i), i' \in \varphi(i)$ denote the set of all dependencies between network i' and its supporting network i . Each dependency is described by a pair (q, r) where $q \in V^{i'} \cup E^{i'}$ and $r \in V^i \cup E^i$. For example, $(p \in V_P^{sc}, j \in V_D^{i1})$ denotes a plant p that needs a power supply and $((p, w) \in E^{sc}, pw \in OD^{i2})$ denotes transportation from plant p to warehouse w that needs telecommunication service between these two locations. The slack variable, s^i , denotes the unmet demand in the supporting network i . Let the binary dependency variable $z^{i',i}$ be an indicator, taking a value of zero when demand of supporting network i is met, but otherwise, one. The following constraint relates the slack variable s^i to the dependency variable $z^{i',i}$ by setting the upper bound of the slack to be the real demand, m^i :

$$s^i \leq m^i * z^{i',i}, \quad (28)$$

If the slack variable is positive, the corresponding dependency variable will be forced to one. Adding this dependency variable $z^{i',i}$ to the capacity constraints of network i' will eventually pass the impact to network i . For example, the flow capacity v is changed to $v^{i'} * (1 - z^{i',i})$. The flow capacity of network i' will be dropped to zero once unmet demand occurs in its supporting network i , i.e., $z^{i',i} = 1$. In addition, adding some fractions to dependency variable z as a multiplier makes the dependency formulation more flexible, modeling a proportional reduction, instead of a complete loss. This construction will be

detailed below, taking power and telecommunication as two typical infrastructures.

3.5.1 Dependency on power

Both plants and distribution centers need electricity to operate. If the power supply is disrupted, production capacity of a plant will drop to zero until power is restored. Likewise, throughput capacity of a warehouse will drop to zero without power supply. Then the company might adjust its production strategy and those plants with power supply may need to produce more products in order to cover the loss of plants caused by the power outage.

A power outage causing an unmet demand is modeled using variable $z_{l,t}^{sc,i_1}, l \in V_D^{i_1}$. It takes a value of zero if node l 's demand is met during time t , but otherwise, one. Occurrence of unmet power demand will drop production capacity of the plant to zero. Such a dependency can be formulated as:

$$\sum_{k \in \phi(p)} pro_{p,k,t} \leq w_p(1 - z_{l,t}^{sc,i_1}), \forall p \in V_P^{sc}, l \in V_D^{i_1}, (p, j) \in DEP(sc, i_1) \text{ and } t \in T \quad (29)$$

where $DEP(sc, i_1)$ is the set of dependency pairs (p, l) , p is the plant and j is the demand node of the power network supporting plant p . If plant p loses power supply, i.e., $z_{l,t}^{sc,i_1}=1$, the right-hand side of constraint (29) will equal zero, which drives the total production to zero.

For warehouses, their throughput capacities will drop to zero without power supply, too. Therefore, these warehouses w cannot receive (30) or ship (31) products:

$$\sum_k \sum_j x_{(j,w),k,t-tl(j,w)}^{sc} \leq q_w(1 - z_{l,t}^{sc,i_1}), \forall w \in v_W^{sc}, l \in V_D^{i_1}, (w, l) \in DEP(sc, i_1) \text{ and } t \in T \quad (30)$$

$$\sum_k \sum_j x_{(w,j),k,t}^{sc} \leq q_w(1 - z_{l,t}^{sc,i_1}), \forall w \in v_W^{sc}, l \in V_D^{i_1}, (w, l) \in DEP(sc, i_1) \text{ and } t \in T \quad (31)$$

3.5.2 Dependency on telecommunication

Sometimes the transportation between plants and distribution centers is affected by telecommunications. Once the telecommunication service fails for any reason, products might not be transported because instructions for supply chain operations are conveyed by telecommunications and shippers will not transport products without instructions. The variable $z_{o,t}^{sc,i_2}$, $o \in OD^{i_2}$ takes a value of one if the communication service of OD pair o is not available during time t , otherwise it is equal to zero. The dependency of transportation capacity on communication is formulated as:

$$\sum_{k \in K} x_{(j,l),k,t}^{sc} \leq v_{(j,l)}(1 - z_{o,t}^{sc,i_2}), \forall (j,l) \in E^{sc}, o \in OD^{i_2}, ((j,l), o) \in DEP(sc, i_2) \text{ and } \forall t \in T \quad (32)$$

where $Dep(sc, i_2)$ is the set of ordered pairs $((j,l), o)$, arc (j,l) is transportation from plant j to warehouse l , and o is the demand pair in the telecommunication network that represents the telecommunication service between these two locations. Failures of communication service for o will drive the total flows on the arc (j,l) in the supply chain to zero.

3.5.3 Dependency within infrastructures

Dependency between telecommunication and power should also be taken into account. Some telecommunication terminals need electric power supply to operate. Without power supply, telecommunication service can't be conveyed through these terminals. Since the supporting power system is modeled as the single commodity network, the dependency variable $z_{l,t}^{i_2,i_1}$, $l \in V_D^{i_1}$ is used in the formulation as follows:

$$\sum_{(j,l) \in E^i} x_{(j,l),t} \leq b_m(1 - z_{l,t}^{i_2,i_1}), \quad \forall m \in V^{i_2}, l \in V_D^{i_1}, (m,l) \in DEP(i', i_1) \text{ and } t \in T \quad (33)$$

where $Dep(i', i_1)$ is the set of ordered pairs (m,l) , m is the telecommunication terminal, and l is the demand node of the power network serving terminal m . If terminal m loses power supply, the capacity of the terminal will be forced to zero.

4 Proposed Problem-solving Process

We investigate a scenario where an extreme event disrupts the power supply, telecommunications, and transportation in a region. Production facilities and warehouses in this area are affected by this event. To address the loss of production and distribution capabilities, managers of the supply chain have the following options: (i) adjust the production at the other plants to cover the loss, (ii) procure products from sources external to the supply chain (sub-contract or out-source), and (iii) re-examine distribution related to the affected region. All these decisions will be influenced by the condition of infrastructure systems. Because of these influences, decisions on the restoration of the services provided by the infrastructure will have an impact on restoration of service for the supply chain. We explicitly study this dependency in this research.

Power and telecommunication infrastructure managers need to develop restoration plans after a disaster. They typically have contractual obligations to restore service in a timely manner, and they need to address the (possibly conflicting) demands of multiple stakeholders. The supply chain manager is one of these stakeholders. We have previously proposed that the infrastructure managers should develop an efficient frontier of solutions to their multi-objective optimization problem [35], and we envision a situation in which the supply chain manager can influence the final selection of a restoration plan from this efficient frontier.

The problem-solving process for the supply chain restoration problem is depicted in Figure 1. Infrastructure managers work out the infrastructure restoration plans, i.e., determine the values of a set of infrastructure restorative decision variable y^i , and provide them to supply chain managers. Then these values enter the supply chain restoration model as input parameters for calculating the best supply chain restoration plan.

Supply chain managers can then provide feedback on the restoration plans to infrastructure managers. For each infrastructure plan, there will be an optimal supply chain plan obtained by our proposed model. Infrastructure managers could have several plans

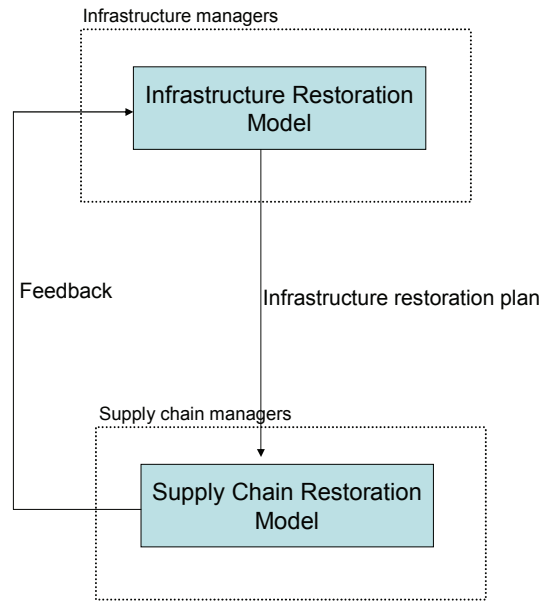


Figure 1: Problem-solving process

that are equally good in terms of restoration of their services. Based on the proposed problem-solving process, supply chain managers can judge, from their perspective, which plan is best, that is, the one that is of the most benefit in restoring their supply chain. The feedback will help infrastructure managers to choose their best plan, perhaps with financial support from supply chain management.

4.1 The supply chain manager’s restoration problem

Generally, supply chain managers seek a set of optimal strategies involving production, inventory, and distribution for a given time horizon T . The supply chain performance could be measured by the total cost, delivery time, quality, flexibility, or revenue. In this paper, the objective of supply chain managers will be minimization of the total cost and unmet demands subject to the limited resources available for restoration, with

consideration of its supporting infrastructures:

$$\begin{aligned}
& \text{minimize } \omega^{sc} \left(\sum_{t \in T} \sum_{(j,l) \in E^{sc}} \sum_{k \in K} c_flow_{(j,l)} x_{(j,l),k,t}^{sc} + \sum_{t \in T} \sum_{p \in V_P^{sc}} \sum_{k \in K} c_pro_{p,k} pro_{p,k,t} \right. \\
& + \sum_{t \in T} \sum_{l \in V_W^{sc} \cup V_P^{sc}} \sum_{k \in K} c_inv_{l,k} inv_{l,k,t} + \sum_{t \in T} \sum_{l \in V_D^{sc}} \sum_{k \in K} c_sl_k s_{l,k,t}^{sc} + \sum_{p \in V_{P'}^{sc}} c_out_p y_p^{sc} \\
& + \sum_{i_1 \in I_1} \omega^{i_1} \sum_{t \in T} \left(\sum_{(j,l) \in E^{i_1}} c_flow_{(j,l)} x_{(j,l),t}^{i_1} + \sum_{j \in V^{i_1}} c_sl_j s_{j,t}^{i_1} \right) \\
& + \sum_{i_2 \in I_2} \omega^{i_2} \sum_{t \in T} \left(\sum_{(j,l) \in E^{i_2}} c_flow_{(j,l)} x_{(j,l),t}^{i_2} + \sum_{o \in OD^{i_2}} c_sl_o s_{o,t}^{i_2} \right) \tag{34}
\end{aligned}$$

The first three items in the objective function are the supply chain costs including transportation, production, and inventory cost, and the fourth item represents the penalty for the slack. Parameter c_sl transfers the unit of the slack into dollars to make it comparable. The fifth item is the cost of outsourcing production to the outside manufacturers. The last four items are infrastructure costs including infrastructure flow costs and the penalty of the slack. Note that parameter ω is used to tradeoff between the supply chain and infrastructures. From the supply chain manager's perspective, ω^i should be much smaller than ω^{sc} and even can be zero to maximize their benefits. Constraints of the supply chain restoration model are those discussed in Section 3, including each individual layer flow constraints and interdependency constraints. A complete supply chain restoration model is presented in Appendix B.

4.2 The infrastructure managers' restoration problem

The infrastructure managers have to construct a restoration plan that meets their contractual obligations, and represents a compromise between multiple objectives including restoring service quickly, minimizing the cost of restoration, and respecting the interests

of various stakeholders. They also have to develop a schedule for restoring various elements of the infrastructures. We give a summary of this problem in this section; for more details see our earlier papers [33],[34],[35]. A Pareto optimal solution to this problem enters the supply chain restoration model as inputs, specifically, the values of parameter y .

The infrastructure managers' problem is inherently a multi-objective one. This opens the possibility of cooperation between these managers and the supply chain manager, with the supply chain manager suggesting alternative Pareto optimal solutions that are more favorable for the supply chain.

5 Computational Results

In order to demonstrate our research on supply chain restoration, we designed a supply chain, based on the following rules, that includes plants, distribution centers, and demand zones all over the United States, as well as an illustrative national power grid (Figure 4) and an illustrative national telecommunication network (Figure 5).

- Location of plant: There are ten plants which together make 400 products, including items to meet urgent needs of victims during a disaster such as bottled water, food, medicines, tents, and so on. Each plant produces approximately 100 products. Production strategies are designed in such a way that the same product can be made by at least two different plants. These two plants are located in different parts of the country to avoid the risk that both of them are affected by the same event.
- Location of distribution center: There are 12 distribution centers across the country and they are located where transportation is convenient.
- Location of demand zone: Demand zones are determined based on geographical location. Generally, they are located near urban areas. Each state has at least one

demand zone. Each zone's demand is proportional to the population of the zone it serves.

- Potential connectivity of the network: Theoretically, a distribution center is able to supply all demand zones and a plant is able to supply all distribution centers and demand zones. Taking transportation into consideration, we assigned distribution centers to the nearest demand zones and assigned a plant to the nearest distribution centers and demand zones, as we show in Figure 7.
- Planning horizon: Restoration requires timely and accurate information and decision making. In this example, the planning horizon is set to ten days and the data such as demands and capacities are based upon one day.

This supply chain, including 10 plants, 12 distribution centers and 400 products, is realistic and comparable to the ones in the real world such as the Kellogg company's supply chain [8] including 5 plants, 7 distribution centers, 15 co-packers, and 80 products, and Pfizer Inc.'s distribution network [7] including 2 large distribution centers, 35 small third-party pool distribution locations. For security reasons, actual national grid data is not available; however, we developed an approximate, "realistic" power grid. Telecommunication network data is from Verizon's telecommunication network.

A scenario of a major disruption in the eastern United States is developed as shown in Figure 6. Communication towers in the Boston area are down and the part of the telecommunication network connecting Boston to other locations (New York City, Buffalo, and Hartford) fails, as shown by the dotted lines in Figure 6. The power supply is disrupted in the shadow areas where a plant, p9, a distribution center, dc12, and the communication terminal of Boston are located. Transportation to the distribution center, dc12, relies on communication between Boston and New York City and thus no products can be shipped to or from dc12. Due to the power outage, both plant p9 and distribution center dc12 have to shut down and the communication of Boston is disrupted.

We assume all needed data, specifically the proposed model parameters, are available to decision makers and are known with certainty. This is a realistic assumption since: (1) sensing and location technology provides infrastructure managers and supply chain managers with data on the condition of the infrastructure or supply chain under their purview; (2) the proposed model implies sharing data and information between infrastructure managers and supply chain managers as the basis for their cooperation, and (3) if damage data are not available, experienced managers can make reasonable estimates which might cause issues of inaccurate data and will be reassessed as data becomes available.

5.1 Infrastructure Restoration

In order to restore the plant in area2, p9, and the distribution center in area3, dc12, infrastructures delivering services in these areas need to be restored first, particularly power in area2 and area3, and communication between Boston and New York City. The restoration option for the power system could be repairing destroyed transmission lines or installing temporary shunts to get power supply from Hartford. In order to restore the telecommunication service, the three destroyed spans need to be repaired and the terminal of Boston needs to obtain sufficient power supply. We use methods mentioned in Section 4.2 to obtain three Pareto-optimal restoration plans for infrastructures, with the integer programs solved to optimality using CPLEX, a commercial package. The results are as follows:

- **Case 1 (cost is the top priority):** The costs include restoration costs and the costs of unmet demand. Repair highlighted transmission lines as in Figure 8 such that power lines in area1 are fixed on the seventh day, area2 on the eighth day, and area3 on the twelfth day; repairing three disrupted spans from Boston such that the span to New York is restored on the third day, Buffalo on the fifth day, and Hartford on the sixth day.

- **Case 2 (time is the top priority):** Unmet demand is driven to zero as quickly as possible, given constraints on repair crews. Repair highlighted transmission lines as in Figure 8 such that power lines in area2 and area3 are fixed on the seventh day and area1 on the sixth day; repairing three disrupted spans from Boston such that the span to New York is restored on the sixth day, Buffalo on the second day, and Hartford on the third day.
- **Case 3 (both time and cost are equally prioritized):** Repair highlighted transmission lines as in Figure 8 such that power lines in area1 are fixed on the sixth day, area2 on the eighth day, and area3 on the fourth day; repairing three disrupted spans from Boston such that the span to New York is restored on the fifth day, Buffalo on the second day, and Hartford on the sixth day.

Infrastructure managers have to trade off between time and cost. The reason why cost and time are chosen as two primary criteria is that emergency response and restoration can be summarized as “saving lives, protecting property and getting life back to normal as soon as possible” [37]. Restoration in our research is focused on “getting life back to normal as soon as possible”, which requires a timely and effectively restoration. As illustrated by the above plans, an earlier restoration would cost a larger amount of money, while a tight budget would sacrifice the pace of restoration. Different sets of priorities give managers multiple options that will enter the supply chain restoration model as inputs.

5.2 Supply Chain Network Restoration

Figure 7 shows the movement of a certain product in the normal situation. Each flow represents a sum of flows over ten days (therefore, it’s not a state on a certain day, but cumulative flows over ten days). For clarity, we present only one product although our model works with 400 products. There are two plants, p6 and p9, which produce this product. Plant p6 is responsible for the demand from the western region while plant

p9 is for the eastern region. After the disruptions occurred, plant p9 and distribution center dc12 were shut down because of failures of power supply and telecommunication service. According to infrastructure managers, it would take around ten days to get disrupted services back to normal. During this period, all the affected areas have to face the problem of a power supply that can only support one area at one time. The company needs a restoration strategy to schedule production, distribution, and inventory during the ten-day period to meet customers' demands.

The supply chain network optimization model gives a solution, namely, let plant p6 produce more products and outsource part of production to the manufacturer represented as plant p11. This "outsource" plant is treated the same as other plants except an additional cost with this plant occurs when outsourcing. In this solution, we didn't consider the infrastructures and so the supply chain was treated as an isolated network.

Taking infrastructures into account, our proposed model gives the following three optimal supply chain restoration strategies corresponding to three different restoration plans for power and telecommunication services in the affected area. In each case, top priority is set to the supply chain, i.e., $\omega^{sc} = 1$, $\omega^{i1} = 0$, $\omega^{i2} = 0$, since supply chain managers are supposed to seek for maximizing their own benefits within the feasible region. The proposed model, including 1,288,740 variables and 432,078 constraints, is written in AMPL and solved by CPLEX 12.5.0.0 on a Lenovo workstation with Intel(R) Core(TM)2 Duo CPU E8400 @ 3.00GHZ, 3.00 GB RAM, running Windows XP Professional. It is a large-scale MILP(Mixed Integer Linear Programming) model and can be solved by Branch-and-Bound algorithm in the reasonable time. The computational performance is shown in Table 1.

- **Case 1:** The limited power supply is delivered to area 1 on the seventh day, and then switched to area 2 from the eighth day. The power outage in area3 lasts for the 10-day restoration. Plant p9 restores its production on the eighth day. For the first seven days, the company outsources most of the required products to plant p11 and plant p6 that can make the similar products as plant p9 increases its production at

Table 1: Computational Performance of the Proposed Model

Instances	Solving Time (Seconds)	Mipgap	MIP Simplex Iterations	Branch and Bound Nodes
Case 1	648.516	1.0e-4	311248	63
Case 2	430.609	1.0e-4	309702	55
Case 3	506.453	4.7e-5	261255	0

a certain cost to cover the remaining part. From the eighth day on, plant p9 restores its production and plant p6 adjusts its production to a low level correspondingly. Since distribution center dc12 is down during the ten-day period due to the power outage, demand zones surrounding dc12 are served by dc11. Therefore, dc12 is isolated from the network.

- **Case 2:** The limited power supply is delivered to area 1 on the sixth day, and then switched to area 2 from the seventh day. The power outage in area3 lasts for the 10-day restoration. Plant p9 is restored on the seventh day. Distribution center dc12 is down during the ten-day period due to the power outage. The strategy is similar to case 1.
- **Case 3:** The limited power supply is delivered to area 3 since the fourth day until the eighth day when it is switched to area 2. Distribution center dc12 is restored on the fifth day when the telecommunication between New York and Boston is restored, and is shut down again from the eighth day when the power supply is reassigned to area 3 so area 2 loses both power and telecommunication services again. Demand zones surrounding dc12 are served by dc11 when dc12 is shut down. Plant p9 restores its production on the eighth day.

Figure 2 presents the supply chain operational costs for ten days for the following scenarios including: the normal situation, operating during the disruption using the existing method, and operating during the disruption using our proposed restoration method. When the disruption occurs the additional costs to restore the supply chain by the existing method will be approximately \$4.22M (the difference in costs between the

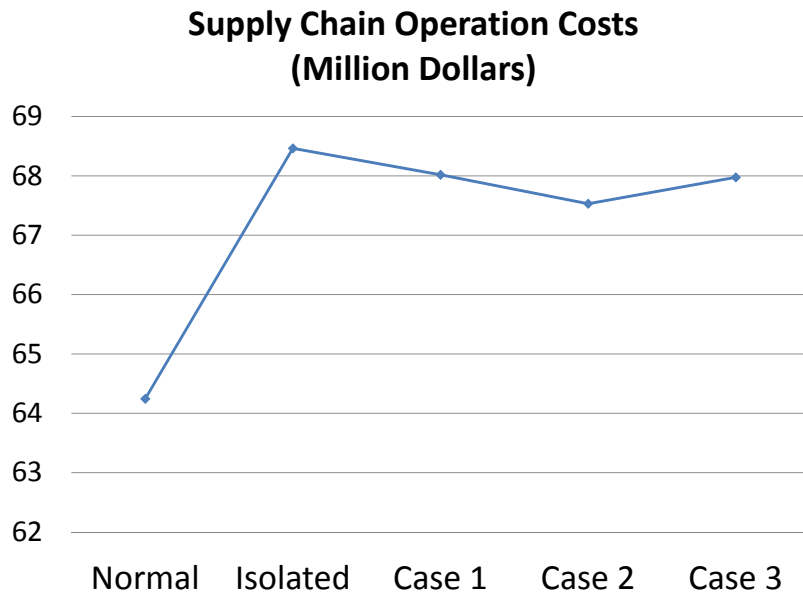


Figure 2: Comparison of Operation Cost Over A 10-day Time Frame

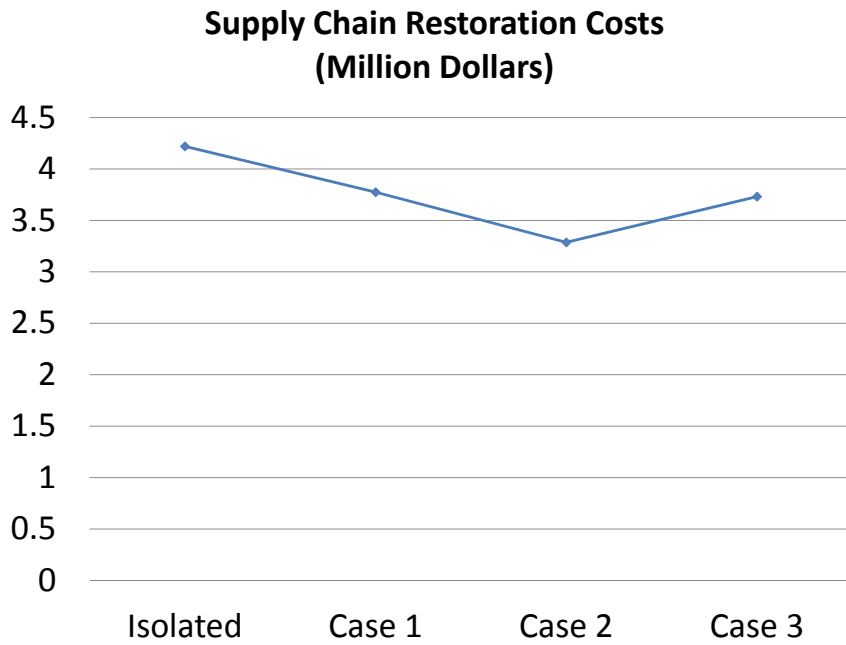


Figure 3: Comparison of Restoration Cost Over A 10-day Time Frame

normal situation and disruption). Our proposed method will save us as much as \$1M (the difference in costs between isolated restoration and case2), over twenty percent of the restoration cost itself. The reduction can be explained by the fact that the more we know about the operational environment of the supply chain, the more accurate decisions we can make. This significant reduction in the restoration cost demonstrates that taking infrastructures into account is not trivial in supply chain restoration. We also find that the earlier the plant is restored, the less the supply chain pays. Case2 costs the least money compared to the other cases, because it restores the disrupted plant and stops higher outsourcing costs the soonest.

When multiple, pareto-optimal, restoration plans is generated, it could be difficult for infrastructure managers to select which of them to implement. By using the proposed approach, supply chain managers are able to access these plans in terms of supply chain restoration costs and then provide recommendations to the infrastructure managers to aid in their decision- making. In this numerical example, case2 is the best plan for the supply chain managers, causing the least supply chain restoration cost. Therefore, case2 would be the recommendation as feedback to infrastructure managers.

As pointed out in the previous section, the problem solving process we propose is an interactive, collaborative one where infrastructure managers first provide their plans to supply chain managers. With infrastructure restoration information and our model, supply chain managers can develop more accurate supply chain restoration plans. Concurrently, supply chain managers can provide feedback to infrastructure managers in terms of costs and/or time for supply chain restoration. As a result, it is possible for infrastructure managers to consider the impact on supply chain restoration as they develop their restoration plan. The proposed problem solving process provides a two-way flow of information between infrastructure managers and supply chains managers, showing the value of cooperation in restoring infrastructures and the supply chain operations that depend on them.

The proposed network model and problem-solving solving process can be viewed as a

public-private partnership in emergency response, providing a theoretical framework for cooperation of supply chain managers and infrastructure managers in emergency management such within an EOC (Emergency Operation Center). Wal-Mart has successfully made the first step, stationing two representatives at the Texas state emergency operations center in time to help the state respond to Hurricane Rita. This cooperation helped Wal-Mart make more informed decisions about where its resources were most needed [38]. The proposed model and process will assist the company with strategies for the complete supply chain, not only distribution, but also production, inventory and transportation.

The core of our approach is to capture interdependencies between infrastructures and the supply chain network and incorporate them into supply chain restoration. To our understanding interdependencies are inherently deterministic and won't change during a certain time. Our approach can model deterministic interdependencies and their impacts on supply chain restoration. Although sensitivity analysis on our deterministic model can deal with uncertainty, we do acknowledge that adding stochasticity makes the model more realistic. However, it would make it much harder to understand impacts of interdependencies or separate interactions between dependencies and stochastic components. Therefore, at this point we focus on deterministic logical relationships, and will put stochasticity into the model in the future work.

6 Conclusions

We present a framework for supply chain restoration which takes into consideration disruptions to the services provided by infrastructure systems; identifies and models the interdependencies between the supply chain network and infrastructures, and uses the model to develop supply chain restoration plans that can improve the company's resilience to disasters. Our goal is an efficient restoration strategy that provides supply chain managers with the ability to work with infrastructure managers. We have shown

that allowing a supply chain manager to suggest a particular Pareto optimal solution to the infrastructure managers can be beneficial to the supply chain. Furthermore, because the solution is Pareto optimal for the infrastructure managers' problem, these managers cannot do better.

However, this is only our first step to incorporating consideration of infrastructures into supply chain restoration decisions. There exist the following limitations to our research and they are also directions for future research:

1. Models discussed in this paper are deterministic and we assume that all needed data are known with certainty. Issues of uncertainty and mitigation will be discussed in our future work.
2. In practice, some negotiation should take place between the sets of managers. The model could be expanded to allow side-payments from the supply chain manager to reduce the cost of restoration of the infrastructures and enable the selection of a restoration plan that is satisfactory to all participants.

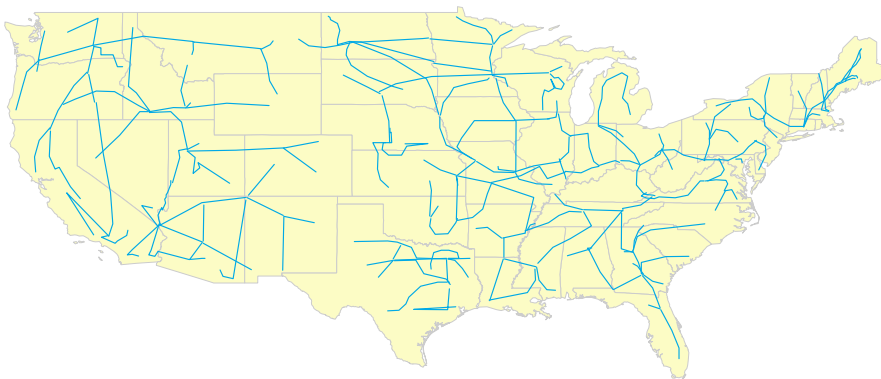


Figure 4: An Approximation to National Power Grid

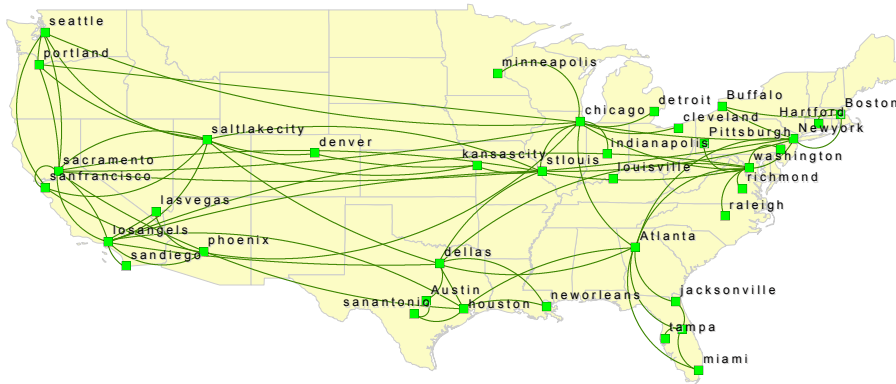


Figure 5: An Approximation to National Telecommunication Network

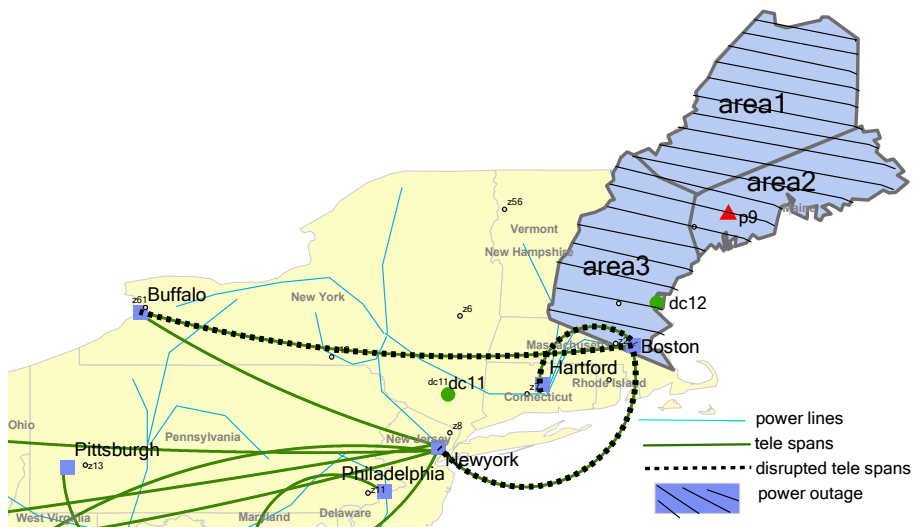


Figure 6: Disruptions

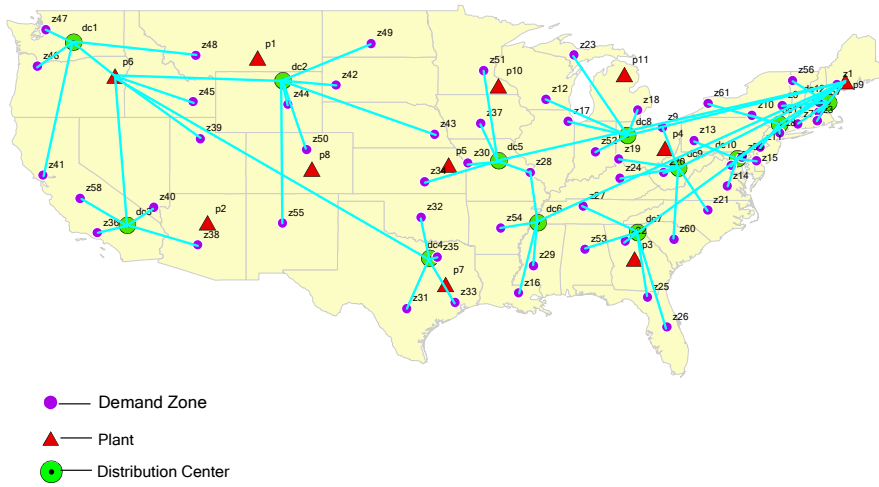


Figure 7: Cumulative Flows of a Certain Product Over 10-day in the Normal Situation

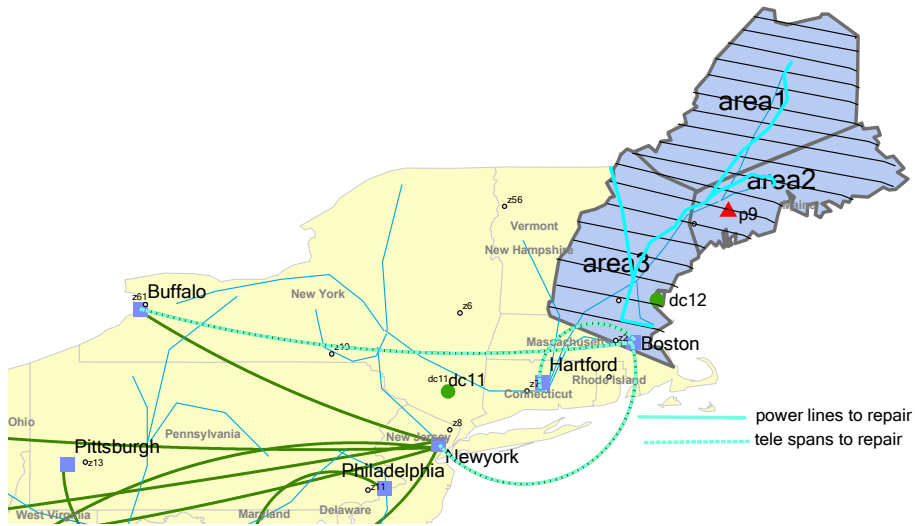


Figure 8: Infrastructure Restoration Plan

7 Acknowledgment

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A Notations

Sets

I : the set of networks.

I_1 : the set of networks modeled as single commodity networks, such as power.

I_2 : the set of networks modeled as multi-commodity networks, such as telecommunication, transportation.

V^i : the set of nodes in network i .

E^i : the set of arcs in network i .

$E^{i'}$: the set of arcs in the event area need to be repaired or replaced.

$\phi(p), p \in V^{sc}$: the set of products produced at plant p .

$\delta(l), l \in V^{sc}$: the set of demand zones supplied by plant (or dc) l .

$\varphi(i), i \in I$: the set of networks that are dependent on network i .

OD^{i_2} : the set of Origin-Destination pairs in multi-commodity network i_2 .

$DEP(i', i)$: the set of all dependencies between network i' and its supporting network i .

Parameters

$v_{(j,l)}, (j,l) \in E^i$: the capacity of arc (j,l) .

$c_flow_{(j,l)}, (j,l) \in E^i$: the flow cost of arc (j,l) .

$b_j, j \in V^i$: the capacity of node j .

T : the time horizon.

$d_{j,k,t}, j \in V_D^{sc}$: the amount of product k required by demand zone j at time t .

$w_p, p \in V_P^{sc}$: the production capacity at plant p .

$q_l, l \in V_P^{sc} \cup V_W^{sc}$: the inventory capacity at location l .

$tl(j,l), (j,l) \in E^{sc}$: the shipping lead time of arc (j,l) .

$c_inv_{l,k}, l \in V_P^{sc} \cup V_W^{sc}$: the inventory cost of product k at location l .

$c_pro_{p,k}, p \in V_P^{sc}$: the production cost of product k at plant p .

$c_out_{p'}, p' \in V_{P'}^{sc}$: the cost of outsourcing production to outside plant p' .

$c_sl_k, k \in K$: the penalty of unmet demand.

$m_j^{i_1}, j \in V^{i_1}$: the demand of node j of the single commodity network i_1 .

$m_o^{i_2}, o \in OD^{i_2}$: the demand of OD pair o of the multi-commodity network i_2 .

$y_{(j,l),t}^i, (j,l) \in E^{i'}$: the parameter indicating whether arc (j,l) of infrastructure i is repaired or installed at time t .

$\omega^i, i \in I$: the decision weight.

Variables

$x_{(j,l),k,t}^{sc}, (j,l) \in E^{sc}$: the amount of commodity k moving on arc (j,l) at time t .

$y_{p',t}^{sc}, p' \in V_{P'}^{sc}$: decision of whether outsourcing work to outside plant p' .

$pro_{p,k,t}, p \in V^{sc}$: the amount of product k produced at plant p at time t .

$inv_{l,k,t}, l \in V^{sc}$: the amount of product k stored at location l at time t .

$s_{j,k,t}^{sc}, j \in V^{sc}$: the slack of product k at demand zone j at time t .

$x_{(j,l),t}^i, (j,l) \in E^i$: the flow on arc (j,l) of infrastructure i .

$z_{l,t}^{i',i_1}, l \in V^{i_1}$: the variable indicating whether the demand of node l of the single commodity network i_1 is met.

$z_{o,t}^{i',i_2}, o \in OD^{i_2}$: the variable indicating whether the demand of OD pair o of the multi-

commodity network i_2 is met.

$s_{j,t}^{i_1}, j \in V^{i_1}$: the unmet demand at node j of the single commodity network i_1 , be restricted to non-negative.

$s_{o,t}^{i_2}, o \in OD^{i_2}$: the unmet demand at OD pair o of the multi-commodity network i_2 , be restricted to non-negative.

$f_{o,n,t}^{i_2}, o \in OD^{i_2}, n \in N$: the flow on the n th path for OD pair o , i.e., from node j to node l , of the multi-commodity network i_2 .

B The complete supply chain restoration model

$$\begin{aligned}
& \text{minimize } \omega^{sc} \left(\sum_{t \in T} \sum_{(j,l) \in E^{sc}} \sum_{k \in K} c_flow_{(j,l)} x_{(j,l),k,t}^{sc} + \sum_{t \in T} \sum_{p \in V_P^{sc}} \sum_{k \in K} c_pro_{p,k} pro_{p,k,t} \right. \\
& + \sum_{t \in T} \sum_{l \in V_W^{sc} \cup V_P^{sc}} \sum_{k \in K} c_inv_{l,k} inv_{l,k,t} + \sum_{t \in T} \sum_{l \in V_D^{sc}} \sum_{k \in K} c_sl_k s_{l,k,t}^{sc} + \sum_{p \in V_{P'}^{sc}} c_out_p y_p^{sc} \left. \right) \\
& + \sum_{i_1 \in I_1} \omega^{i_1} \sum_{t \in T} \left(\sum_{(j,l) \in E^{i_1}} c_flow_{(j,l)} x_{(j,l),t}^{i_1} + \sum_{j \in V^{i_1}} c_sl_j s_{j,t}^{i_1} \right) \\
& + \sum_{i_2 \in I_2} \omega^{i_2} \sum_{t \in T} \left(\sum_{(j,l) \in E^{i_2}} c_flow_{(j,l)} x_{(j,l),t}^{i_2} + \sum_{o \in OD^{i_2}} c_sl_o s_{o,t}^{i_2} \right) \tag{35}
\end{aligned}$$

$$pro_{p,k,t} + inv_{p,k,t-1} = \sum_{l \in V_W^{sc} \cup V_D^{sc}} x_{(p,l),k,t}^{sc} + inv_{p,k,t}, \quad \forall p \in V_P^{sc}, \forall k \in \phi(p) \text{ and } t \in T \tag{36}$$

$$\sum_{p \in V_P^{sc}} x_{(p,w),k,t-tl(p,w)}^{sc} + inv_{w,k,t-1} = \sum_{l \in V_D^{sc}} x_{(w,l),k,t}^{sc} + inv_{w,k,t}, \quad \forall w \in V_W^{sc}, k \in K \text{ and } t \in T \tag{37}$$

$$\sum_{w \in \delta(l)} x_{(w,l),k,t-tl(w,l)}^{sc} + \sum_{p \in \delta(l)} x_{(p,l),k,t-tl(p,l)}^{sc} + s_{l,k,t}^{sc} = d_{l,k,t}, \forall l \in V_D^{sc}, \forall k \in K \text{ and } t \in T \quad (38)$$

$$\sum_{k \in K} x_{(j,l),k,t}^{sc} \leq v_{(j,l)}, \quad \forall (j,l) \in E^{sc} \text{ and } t \in T, \quad (39)$$

$$\sum_{k \in \phi(p)} pro_{p,k,t} \leq w_p, \quad \forall p \in V_P^{sc} \text{ and } t \in T, \quad (40)$$

$$\sum_{k \in \phi(p)} pro_{p,k,t} \leq w_p * y_{p,t}^{sc}, \quad \forall p \in V_{P'}^{sc} \text{ and } t \in T, \quad (41)$$

$$\sum_{k \in K} inv_{l,k,t} \leq q_l, \quad \forall l \in V_P^{sc} \cup V_W^{sc} \text{ and } t \in T. \quad (42)$$

$$\sum_{l \in V_W^{sc} \cup V_D^{sc}} x_{(p,l),k,t}^{sc} = 0, \quad \forall p \in V_P^{sc}, k \in K \setminus \phi(p), \text{ and } t \in T. \quad (43)$$

$$\sum_{(j,l) \in E^{i_1}} x_{(j,l),t}^{i_1} \leq b_j, \quad \forall j \in V_S^{i_1} \text{ and } t \in T \quad (44)$$

$$\sum_{(l,j) \in E^{pr}} x_{(l,j),t}^{i_1} + s_{j,t}^{i_1} = m_j^{i_1}, \quad \forall j \in V_D^{i_1} \text{ and } t \in T \quad (45)$$

$$s_{j,t}^{i_1} \leq m_j^{i_1} * z_{j,t}^{i,i_1}, \quad \forall j \in V_D^{i_1}, i \in \varphi(i_1), t \in T \quad (46)$$

$$\sum_{(i,j) \in E^{i_1}} x_{(i,j),t}^{i_1} - \sum_{(j,l) \in E^{i_1}} x_{(j,l),t}^{i_1} = 0, \quad \forall j \in V_T^{i_1} \text{ and } t \in T \quad (47)$$

$$x_{(j,l),t}^{i_1} \leq v_{(j,l)} * y_{(j,l),t}^{i_1}, \quad \forall (j,l) \in E^{i_1'} \text{ and } t \in T \quad (48)$$

$$x_{(j,l),t}^{i_1} \leq v_{(j,l)}, \quad \forall (j,l) \in E^{i_1} \setminus E^{i_1'} \text{ and } t \in T \quad (49)$$

$$\sum_{n \in 1..N} \sum_{(o,n) \text{ contains } (j,l)} f_{o,n,t}^{i_2} = x_{(j,l),t}^{i_2}, \quad \forall (j,l) \in E^{i_2} \text{ and } t \in T \quad (50)$$

$$\sum_{n \in 1..N} f_{o,n,t}^{i_2} + s_{o,t}^{i_2} = m_o^{i_2}, \quad \forall o \in OD^{i_2} \text{ and } t \in T \quad (51)$$

$$s_{o,t}^{i_2} \leq m_o^{i_2} * z_{o,t}^{i,i_2}, \quad \forall o \in OD^{i_2}, i \in \varphi(i_2) \text{ and } t \in T \quad (52)$$

$$\sum_{(l,j) \in E^{i_2}} x_{(l,j),t}^{i_2} \leq b_j, \quad \forall j \in V^{i_2} \text{ and } t \in T \quad (53)$$

$$x_{(j,l),t}^{i_2} \leq v_{(j,l)} * y_{(j,l),t}^{i_2}, \quad \forall (j,l) \in E^{i_2'} \text{ and } t \in T \quad (54)$$

$$x_{(j,l),t}^{i_2} \leq v_{(j,l)}, \quad \forall (j,l) \in E^{i_2} \setminus E^{i_2'} \text{ and } t \in T \quad (55)$$

$$\sum_{k \in \phi(p)} pro_{p,k,t} \leq w_p(1 - z_{l,t}^{sc,i_1}), \quad \forall p \in V_P^{sc}, l \in V_D^{i_1}, (p,j) \in DEP(sc, i_1) \text{ and } t \in T \quad (56)$$

$$\sum_k \sum_j x_{(j,w),k,t-tl(j,w)}^{sc} \leq q_w(1 - z_{l,t}^{sc,i_1}), \quad \forall w \in v_W^{sc}, l \in V_D^{i_1}, (w,l) \in DEP(sc, i_1) \text{ and } t \in T \quad (57)$$

$$\sum_k \sum_j x_{(w,j),k,t}^{sc} \leq q_w(1 - z_{l,t}^{sc,i_1}), \quad \forall w \in v_W^{sc}, l \in V_D^{i_1}, (w,l) \in DEP(sc, i_1) \text{ and } t \in T \quad (58)$$

$$\sum_{k \in K} x_{(j,l),k,t}^{sc} \leq v_{(j,l)}(1 - z_{o,t}^{sc,i_2}), \quad \forall (j,l) \in E^{sc}, o \in OD^{i_2}, ((j,l), o) \in DEP(sc, i_2) \text{ and } \forall t \in T \quad (59)$$

$$\sum_{(j,l) \in E^i} x_{(j,l),t} \leq b_m(1 - z_{l,t}^{i',i_1}), \quad \forall m \in V^{i_2}, l \in V_D^{i_1}, (m,l) \in DEP(i', i_1) \text{ and } t \in T \quad (60)$$

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