MULTIVEHICLE ROUTING WITH PROFITS AND MARKET COMPETITION

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ABSTRACT

This paper deals with a multiple vehicle routing problem in which profit is maximized subject to competition. This problem will be referred to as the multiple vehicle routing problem with profits and competition (MVRPPC). The MVRPPC differs from traditional multivehicle routing problems in three ways: (1) competition is incorporated into the process, (2) the objective is to maximize profits rather than minimize costs, and (3) it is assumed that trucks leave and return to their home bases empty, thus any freight picked up in a tour must be delivered in that same tour (which represents the case of for-hire carriers). The solution method takes a "cluster first, route second" approach in which the clustering phase combines a geometric clustering with a generalized assignment problem (GAP). The routing is performed using a tabu search. To get an idea of how well the tabu search performs, an alternative method for routing was developed which consisted of a mixed integer program (MIP) based on the flow formulation of the traveling salesman problem. The solution approach was applied to a series of problems of varying size and complexity with the routing performed by both the tabu search and the MIP formulations. A comparison of the tabu search and MIP solutions indicated that the tabu search solutions were practically the same than the corresponding MIP solutions, with tabu search objective function values which were no more than 0.70% of the MIP values. As an illustration of the potential uses of the methodologies developed, the paper analyzes the role of the degree of market transparency on the geographic segmentation of the market.

INTRODUCTION

The multiple vehicle routing problem with profits and competition (MVRPPC) represents an extension of the multivehicle routing problem in that it: (1) incorporates competition into the routing process, (2) maximizes profits rather than minimizes costs, and (3) assumes that trucks leave and return to their home bases empty, thus any freight picked up in a tour must be delivered in that same tour.

The MVRPPC has potential real-world applications as it enables the study of strategic competition among private trucking companies in such areas as the transportation of aggregates (e.g., sand, gravel) to construction sites. This case involves a set of production sites where the aggregates are processed and a set of demand nodes which are the construction sites.

While no previous research was found in which competition was incorporated into the routing process, research has been done on two types of problems which involve routing based on profit maximization. The first type of problem is the merchant subtour problem which involves a merchant who buys commodities where they are cheap and transports them to cities where he can sell them at a profit (Verweij et al, 2003). The problem is to decide which demand cities to visit and in what order in order to maximize his profits. This problem differs from the MVRPPC in that it involves only one company, rather than several competing companies, and it deals with intercity, rather than urban, freight movements. The second type of routing problem involving profit maximization is the traveling salesman problem with profits (Feillet et al, 2005). This is a generalization of the traveling salesman problem in which a profit is made when a vertex is visited and there is no requirement that all vertices be visited. A multiple vehicle version of this problem, developed and applied to freight transportation in Feillet, (2001), dealt with freight movements between plants in the car industry thus it did not include the requirement that each tour begin and end at a home base.

Similarly, no literature was found in which all the freight picked up in a tour had to be delivered in that tour. Vehicle routing problems generally fall into one of three categories: (1) pick up or delivery only, (2) pick up or delivery only followed by an optional backhaul, and (3) combined pick up and delivery (Assad, 1988) In the combined pick up and delivery case, the pick ups and deliveries are either paired (Nanry, et al., 2000) or deliveries are made before pick ups (line haul-back haul) (Jacobs-Blecha, et al.,1993). The case considered in this paper, in which all cargo is delivered in the tour, is very important for urban goods modeling because approximately half the cargoes transported in the United States are transported by common carriers (USDOT, 2002), which are the ones that tend to follow the pick-up/delivery patterns discussed here.

Another important consideration is that the MVRPPC is a crucial component of Spatial Price Equilibrium formulations of urban goods with explicit consideration of actual logistic practices. In this context, the MVRPPC enables the attainment of an approximation of the Spatial Price Equilibrium solution taking into account commercial vehicle trip chaining behavior (see for instance, Holguín-Veras, 2000). Among other things, these novel methodologies enable the modeling of urban goods markets with complete consistency with the underlying economic dynamics, while explicitly considering trip chaining behavior. The latter point is important because the Spatial Price Equilibrium formulations to date (e.g., Samuelson (1952), Takayama and Judge (1964, 1971), Friesz et al.,1986)—though of undeniable value—are based on network flow formulations that are unable to consider trip chains. In providing the analytical core of a urban goods formulation based on Spatial Price Equilibrium, the MVRPPC extends this important field to the realm of discrete mathematics.

This paper has three sections in addition to this introduction. *Methodology* describes the solution method and *Results and Discussion* presents results for a series of sample problems. Finally, *Conclusions* summarizes the conclusions regarding the algorithmic results.

METHODOLOGY

Before discussing the details of the formulations developed in the paper, it is important to conceptually describe the problem at hand. Among other things, this would enable the reader to get a clear idea about what the paper is trying to accomplish and its significance to freight demand modeling.

A very simple example of the problem which this paper addresses is shown in Figure 1. As shown in this figure, each company operates from a home base from which trucks leave empty at the start of a tour and to which they return empty at a tour's end. Within a tour, trucks pick up freight from production nodes and deliver freight to attraction nodes. Since some production and attraction nodes are available to more than one company, they will initially be included in the tours of more than one company. These contested nodes are awarded to the company which can service them at lowest cost through an iterative process so that, in the final solution, all nodes are serviced by exactly one company. Without any loss of generality, the production costs at the production nodes are assumed to be constant. An implicit assumption is that the cargo being transported corresponds to a generic commodity. These assumptions were made to focus on the role of transportation costs.

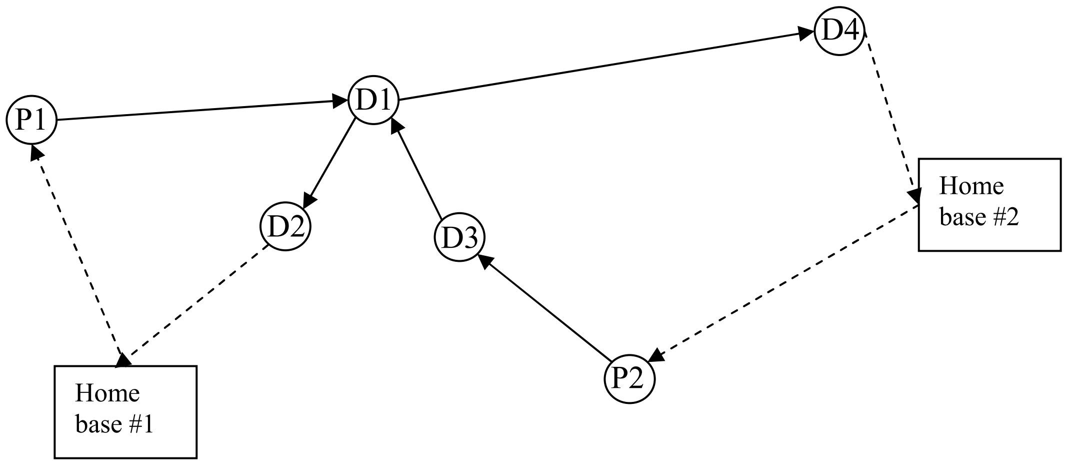


Figure 1: An example of the problem to be solved

Note: The trucking companies' home bases are denoted by rectangles. The tours are denoted with arrows and numbered stops. Dashed arrows indicate empty trips and solid arrows indicate loaded trips. Production nodes are denoted by a "P" and attraction nodes by a "D".

The solution method outlined in this paper combines a number of well-established mathematical programming and meta-heuristic techniques to solve the network routing problem within a cluster first, route second framework. The cluster phase combines a geometric clustering with a generalized assignment problem and the routing phase is performed by a tabu search. The solution approach begins with a geometric clustering based on an estimated tour time. This geometric clustering provided an estimate of the cost of including node *i* in cluster *j*. Then, these cost coefficients are used in a generalized assignment problem which incorporated constraints insuring tour feasibility to produce minimum cost clusters which can be turned into feasible tours. This approach is very similar to that developed in Nygard, et al., (1988).

Once feasible node clusters were obtained, the routing was performed by tabu search. After an initial tabu search solution was obtained, the tours for each player were examined node by node to see if there were any exchanges of two nodes in different tours of the same player which would increase the profit for at least one tour and not decrease the profit for any tour. After all possible node exchanges were examined, all nodes which received bids from more than one company were identified. The delivery costs for each bidding company were then calculated and these nodes were removed from the node sets of players whose bids were not the lowest and the clustering/routing procedure was repeated with the modified node sets. This process was continued until each node was included in exactly one tour resulting in an approximation of spatial price equilibrium. This cluster first, route process is outlined in Figure 2. Before describing the details of the clustering and routing procedures in the subsequent sections, the incorporation of competition into the routing process is presented in the next section.

Figure 2: The cluster first, route second procedure

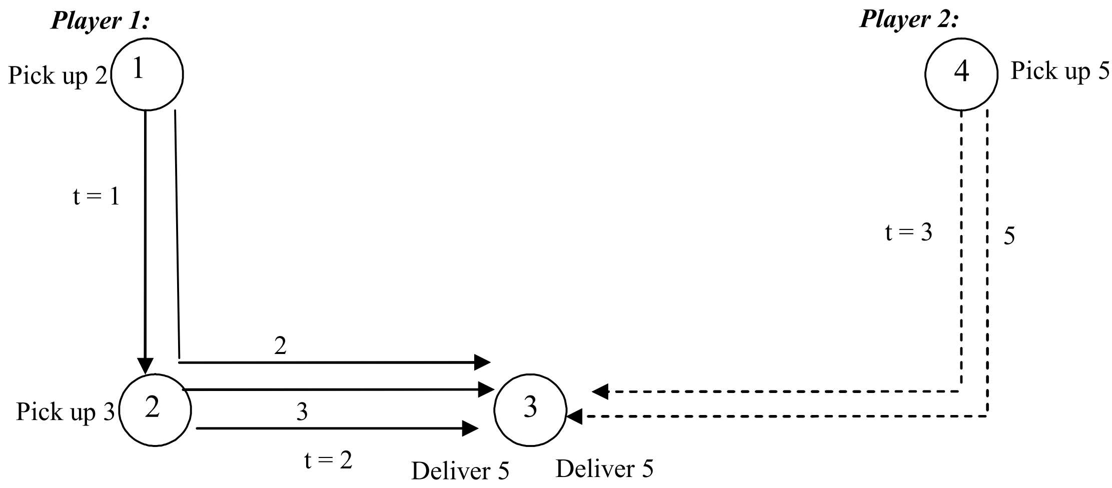
Incorporation of competition into the routing process

An important component of this research is the explicit consideration of competition and the level of market transparency, which are two key elements that are intertwined. Market transparency refers to the level of awareness which competing carriers have of the cargoes to be transported. In a perfectly transparent market, unattainable in real life, all carriers know about all the cargoes to be transported, which translates into an environment of perfect competition and marginal cost pricing. In this context, one would expect a certain degree of geographic segmentation of the market. At the other end of the spectrum, only one carrier knows about the cargoes that require transport, leading to a situation in which carriers could impose price differentiation on their customers (see Holguín-Veras and Jara-Díaz, 1999). This environment is bound to lead to longer and more inefficient tours. In all cases, competition is introduced by specifying a subset of randomly selected nodes that are available to more than one company. In this context, nodes available to only one company represent production or attraction nodes whose existence is only known to that company. Since there is no information about the level of market transparency in real life, it was simulated in the modeling process by assuming different levels of the *degree of market transparency* (denoted by ρ), which represents the fraction of production and demand nodes available to more than one carrier with respect to the total number of nodes. The company which is able to service these nodes at lowest cost wins the bid to service them. Transportation costs are calculated as follows:

$$c_i = c_{orig} \cdot |subtour_i| + c_{travel} \cdot t(subtour_i) + c_{wait\_time} \cdot freight(subtour_i) \quad (1)$$

Where i is a contested node, $subtour_i$ is the set of stops from which freight is picked up to be delivered to i or to which freight picked up at i is delivered, $|subtour_i|$ is the number of nodes in $subtour_i$, c_{orig} is a flat fee charged for each stop, c_{travel} is the travel cost, and c_{wait_time} is the cost due to loading and unloading of freight, and *freight(subtour)* is the freight picked up and delivered in $subtour_i$.

For each contested node i, the company with the lowest c_i wins the bid to service that node and the losing companies lose the opportunity to service that node. In the event of a tie, the node remains in the node sets of the companies with tying bids and these companies get another chance to service the node. A very simple example of two companies competing to service a node is shown in Figure 3.



Where:

t = travel time

$$c_i = c_{orig} \cdot |subtour_i| + c_{travel} \cdot t(subtour_i) + c_{wait\_time} \cdot freight(subtour_i) \quad (1)$$

Assuming for illustration purposes that c_{orig} = \$10, c_{travel} = \$4 per hour and c_{wait_time} = \$2 per unit freight:

c_3 for player 1: c_3 = \$10 x 2 + \$4 x (t_{12} + t_{23}) + \$2 x 5
= \$20 + \$4 x 3 + \$2 x 5 = \$42.00

c_3 for player 2: c_3 = \$10 x 1 + \$4 (t_{43}) + \$2 x 5
= \$10 + \$4 x 3 + \$2 x 5 = \$32.00

Figure 3: Calculation of transportation costs for servicing a contested node

Radial sweep/GAP clustering

Of the various clustering methods examined, it was found that a radial sweep method was preferable in that it was easy to control both the number of clusters and the number of nodes in each cluster by adjusting the tour time limit. In the radial sweep method of Gillett and Miller (1974), the polar coordinates of each stop are calculated with the radius defined as the distance between the home base and the stop and the angle defined by two rays – one from the home base to some arbitrary point and the other from the home base through the stop. The stops are sorted according to the size of their polar-coordinate angle, where ties are broken by the distance to the home base. Then a sweep is performed which partitions the stops into routes beginning with the stop that has the smallest angle and adding stops until the estimated total travel time exceeds the tour time duration limit. The violating stop then becomes the first stop in the next route. This process is continued until all stops are assigned to a cluster.

The end result of the radial sweep is a partition of nodes into clusters in which the nodes are relatively close to each other. However, these sets of nodes cannot be turned into feasible routes because the total productions and attractions of the nodes in each cluster do not necessarily balance. Thus, the final clustering is performed using the GAP presented in Nygard et al., (1988) with several additional constraints which is as follows:

$$\min \sum_{j \in J} \sum_{k \in K} c_{kj} x_{kj} \tag{2}$$

subject to:

$$\sum_{k \in K} x_{kj} = 1 \qquad \text{for all } j \in J \tag{3}$$

$$\sum_{j \in J} p_j x_{kj} - \sum_{j \in J} a_j x_{kj} = 0 \qquad \text{for all } k \in K \tag{4}$$

$$\sum_{j \in J} p_j x_{kj} \geq \alpha \sum_{j \in J} p_j \qquad \text{for all } k \in K \tag{5}$$

$$\sum_{j \in J} p_j x_{kj} \leq \beta \sum_{j \in J} p_j \qquad \text{for all } k \in K \tag{6}$$

$$x_{kj} \in \{0,1\} \qquad \text{for all } k \in K, j \in J \tag{7}$$

Where K is the set of vehicles, J is the set of stops, c_{kj} is the cost of assigning stop j to vehicle k, x_{kj} is a binary variable equal to one if stop j is assigned to vehicle k, p_j and a_j are the production and attraction of node j respectively, and α and β are parameters equal to $\alpha < \frac{\sum_j p_j}{|K|}$ and $\beta > \frac{\sum_j p_j}{|K|}$. Constraint (3) insures that each node is assigned to exactly one cluster. Constraint (4) insures that the total productions and attractions of each cluster are equal. Constraints (5) and (6) set minimum and maximum cluster capacities so that the amount of freight in each cluster is relatively uniform.

In order for the GAP to produce clusters which can be turned into high quality tours, the cost coefficients in the objective function must accurately reflect the cost of including stop j in route k. Since the actual tours are not known yet, this cost is not known and must be estimated. In Nygard et al., (1988), the cost of adding stop *j* to vehicle *k*'s tour was estimated as the difference between: (1) the cost of visiting node *j*, then visiting cluster *k* (as represented by its centroid), and returning to the home base, and (2) the cost of a round trip from the home base to node *j* and back to the home base. The cost incurred is then:

$$c_{kj} = d_{hb,j} + d_{j,k} - d_{k,hb} \tag{8}$$

Where $d_{hb,j}$ is the distance from the home base to node *j*, $d_{j,k}$ is the distance from the node *j* to the centroid of the *k*th cluster, and $d_{k,hb}$ is the distance from the home base to centroid *k*.

This method tended to produce reasonable cost coefficient estimates for nodes which were farther from the home base than the cluster centroid, but not very good values for nodes which were closer in. An alternative method is as follows:

$$c_{kj} = d_{nearest\_node(k),j} + d_{j,nearest\_node(k)} \tag{9}$$

Where $d_{nearest\_node(k),j}$ is the distance between node *j* and the node in cluster *k* nearest to *j*.

This alternative method appeared to produce more reasonable cost coefficient estimates regardless of where the node was located and, since the nearest node in cluster *k* is node *j* itself if it is already in cluster *k*, there is zero cost for keeping nodes in their current cluster. Once clusters of nodes which can be turned into feasible tours were obtained, the routing was performed with a tabu search which will be described next.

Tabu search formulation for routing

Tabu search is a local search method for combinatorial optimization problems (Glover, 1986). As described in Glover and Laguna (1993), it explores the solution space by moving from a solution x_i at iteration *i* to the best solution x_{i+1} in a subset of the neighborhood $N(x_i)$ of x_i. x_{i+1} does not necessarily improve on x_i and a tabu list is maintained to prevent the search from cycling over a sequence of solutions. The tabu list keeps track of some attributes of previous discovered solutions and any new solution with these attributes is considered tabu for *t* iterations. The neighborhood $N(x_i)$ of x_i is a set of solutions that can be reached from x_i by specified moves. A very common move used in routing problems is called a λ-interchange in which up to λ customers are exchanged between two routes. The attributes of such a move are often the edges which are removed from and added to the routes (Hjorring, 1995). The tabu status of a move can be revoked if it meets an aspiration criterion, for example, the move results in a solution which exceeds any previously discovered solution.

As mentioned above, one of the ways in which the routing problem discussed in this paper differs from the traditional routing problem is that the objective is to maximize profits, rather than to minimize costs. The profit function to be maximized in the tabu search is the following:

$$profit = C_W \sum_{i=1}^{N} (p_i + a_i) - C_T \sum_{(i,j) \in Tour} t_{i,j} \tag{10}$$

Where N is the number of nodes in the tour, C_W is the benefit of picking up and delivering freight, C_T is the travel time cost, p_i and a_i are the productions and attractions at node i, and $t_{i,j}$ is the travel time from node i to node j.

In this function, profit is the difference between: (1) the benefit from picking up and delivering freight and (2) the costs incurred in traveling from node to node to pick up and deliver freight.

The following subsections describe important characteristics of the tabu search formulation developed for solving the routing problem of concern to this paper, including the tabu search moves, the tabu list, and a general outline of the tabu search algorithm.

Tabu search moves

The tabu search formulation uses four types of moves – adding 2 nodes (one production and one attraction) to the current tour (*add2*), swapping two nodes (one of each) that are in the tour with two that are not (*swap2*), adding one node (*add_node*), and swapping one node (*swap_node*). The decision of which move to make and, for the moves involving one node, which type of node to process, is based on the current tour's production and attraction potential. This quantity is the total difference between the demands of the nodes in the tour and the amount of freight actually picked up/delivered at those nodes:

$$\Delta_{PROD} = \sum_{i} (p_i - u_i) \quad \Delta_{ATT} = \sum_{i:b_i<0} (a_i - d_i) \tag{11}$$

Where u_i and d_i are the amounts of freight currently being picked up from or delivered to node *i*.

If $\Delta_{PROD} = \Delta_{ATT}$, then an *add_2* move is made. If the best possible *add_2* move results in a violation of the tour time constraint, then a *swap_2* move is made. If $\Delta_{PROD} \neq \Delta_{ATT}$, then an *add_node* move is made. If the best possible *add_node* move results in a violation of the tour time constraint, then a *swap_node* move is made.

In the *add_2* and *swap_2* moves, both a production and an attraction node are added/swapped. In the *add_node* and *swap_node* moves, on the other hand, a decision must be made as to whether a production node or an attraction node should be added or swapped. If $\Delta_{PROD} > \Delta_{ATT}$, then the tour has more "unused" production, so an attraction node should be added or swapped. Otherwise, the tour has more "unused" attraction and a production node is added or swapped.

Tabu list

A tabu list keeps track of the arcs that are added to and removed from the tour as a result of these moves. The tabu list is a three dimensional array, *TABU(i, j, k)* where *i* and *j* represent the arc *(i,j)* and *k* can have a value of 1 which indicates that the arc is added to the tour or a value of 2 which indicates that the arc is removed from the tour. These arcs remain on the tabu list for *p* iterations.

For each move, the set of possible moves is identified and the move which results in the highest objective function value is selected. If this move does not involve arcs that are on the tabu list, it is accepted and the tour and tabu list are updated. If the move does involve tabu arcs but the resulting objective function value is greater than any previously attained (the aspiration level, *AL*), it is accepted, the tour and tabu list are updated, and the resulting objective function value becomes the new aspiration level. This process is repeated until a specified number of iterations are performed without any improvement in the best found solution. Thus, the stopping criterion is that y iterations have been performed without finding a better solution. As a practical matter, $y = 10$ appeared to be a sufficiently large number of iterations. A flow chart of the tabu search solution process is shown in Figure 4.

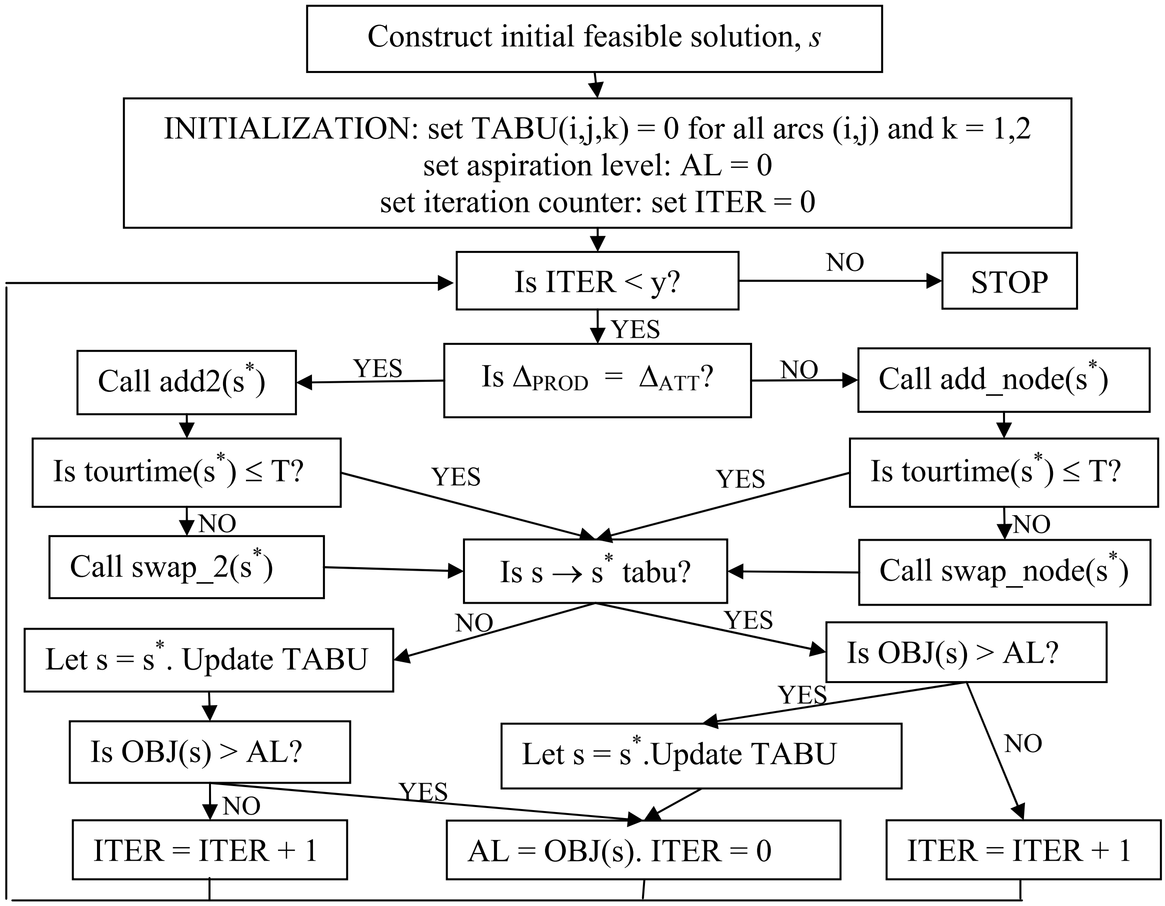


Figure 4: A flow chart of the tabu search

After the tabu search procedures outlined in this subsection were applied in the routing phase of the solution process, the resulting tours were examined to see if there were node exchanges that could be made which would increase the profit of at least one tour without decreasing the profit of the other. This node exchange process is described in the following section.

Intra-player node exchanges

For each node in the tours constructed in the tabu search, the nodes in the other tours for the same player were scanned to identify nodes which had the same production or attraction. Once a pair of nodes in two different tours with the same freight demand was identified, the nodes were exchanged and each was inserted in the new tour in the most profitable location. The new profit for each tour was calculated and, if this new profit was at least equal to the current profit for both tours, then the exchange was accepted. The rationale for this step is that the clustering/GAP/routing solution procedure breaks down the problem so that each tour is constructed independently and this exchange process allows the solution procedure to examine the tours of each player together to see if the set of tours can be made more profitable.

Each step in the clustering/GAP/routing procedure has been described. In order to get some idea about the quality of the resulting solutions, the same procedure was undertaken with the routing solved by a mixed integer program which is based on the flow formulation of the traveling salesman problem. This formulation is described in the next section.

Mixed integer programming formulation

The starting point for this approach was the flow formulation of the traveling salesman problem (Ahuja et al., 1993). The parameters of the problem are as follows:

N – number of nodes

A – the arc set

C_T – cost of travel time

C_W – cost of loading and unloading freight

C_P – cost for not visiting a node

T – maximum allowable tour time

t_{ij} – travel time between node i and node j based on city block distance and constant travel speed

t_W – time it takes to load or unload one unit of freight

b_i – freight demand at node i ($b_i > 0$ means node i has freight to be picked up and delivered elsewhere, $b_i < 0$ means node i needs freight to be delivered to it)

Q – capacity of a truck

The variables are:

x_{ij} – a binary variable indicating whether arc *ij* is on the tour

z_{ij} – the flow on arc *ij* which represents the amount of freight on the truck from node *i* to node *j*

u_i – the amount of freight picked up at node *i*

d_i – the amount of freight delivered to node *i* (constrained to be nonpositive since, for flow conservation purposes, attractions are negative)

f_i – a penalty for not including node *i* in the tour

The problem can be stated as follows:

$$\max\left(-\sum_{i,j=1}^{N} C_T t_{ij} x_{ij} + \sum_{i=1}^{N} C_W u_i - \sum_{i=1}^{N} C_W d_i - \sum_{i=1}^{N} C_P f_i\right) \quad (12)$$

subject to:

$$\sum_{i,j=1}^{N} t_{ij} x_{ij} + \sum_{i=1}^{N} t_W u_i - \sum_{i=1}^{N} t_W d_i \le T \quad (13)$$

$$\sum_{j=\{1,\ldots,N\}\setminus\{i\}} x_{ji} + f_i = 1 \qquad \text{for i} = 1, \ldots, N \quad (14)$$

$$\sum_{j=\{1,\ldots,N\}\setminus\{i\}} x_{ij} + f_i = 1 \qquad \text{for } i = 1, \ldots, N \quad (15)$$

$$\sum_{i=1}^{N} x_{1i} = 1 \quad (16)$$

$$\sum_{i=1}^{N} x_{i1} = 1 \quad (17)$$

$$\sum_{\{j:(i,j)\in A\}} z_{ij} - \sum_{\{j:(j,i)\in A\}} z_{ji} = u_i \qquad \text{for } i : b_i > 0 \quad (18)$$

$$\sum_{\{j:(i,j)\in A\}} z_{ij} - \sum_{\{j:(j,i)\in A\}} z_{ji} = d_i \qquad \text{for } i : b_i < 0 \quad (19)$$

$$z_{ij} - Qx_{ij} \le 0 \quad \text{for } (i, j) \in A \quad (20)$$

$$u_i \le b_i \quad \text{for } i : b_i > 0 \quad (21)$$

$$d_i \ge b_i \quad \text{for } i : b_i < 0 \quad (22)$$

$$d_i \le 0 \quad \text{for } i : b_i < 0 \quad (23)$$

$$\sum_{i=1}^{N} u_i + \sum_{i=1}^{N} d_i = 0 \quad (24)$$

$$\sum_{(i,j)\in C} x_{ij} \le |C| - 1 \qquad \text{for } 2 \le |C| \le N - 1 \quad (25)$$

$$x_{ij} \in \{0,1\} \qquad \text{for i} = 1, \ldots, N, \text{ for j} = 1, \ldots, N \quad (26)$$

$$f_i \in \{0,1\} \qquad \text{for i} = 1, \ldots, N \quad (27)$$

Where *C* is a cycle which is not a maximal subtour starting and ending at the home base.

The objective function in (12) maximizes total pickups and deliveries while minimizing travel time. The first term is the travel time cost for the tour and the last two terms represent the benefits for picking up and delivering freight. The tour time constraint in (13) insures that the duration of the tour does not exceed the time limit T. The first term represents the travel time and the last two terms represent the loading and unloading time. The next group of constraints in (14) – (17) insures that the tour begins and ends at the truck's home base which is designated as node 1 and that each node that is visited is visited exactly once. The f_i term in (14) and (15) allows the tour to skip a node if there is not enough time to visit it.

Since this formulation does not require that each node's freight demands are met exactly, the flow conservation equality constraints in (18) and (19) involve the amount of freight that is actually picked or delivered rather than the amount that is available at each node. The next group of constraints in (20) insures that only the arcs on the tour have flow, that is, if x_{ij} is zero then so is z_{ij}. The constraints in (20) also insure that the truck's capacity is not exceeded.

The constraints in (21) – (24) deal with the amount of freight picked up and delivered. The constraints in (21) and (22) insure that the amount of freight picked up at a production node does not exceed the amount of freight available at that node and the amount of freight delivered at an attraction node does not exceed the amount of freight that node needs. The constraints in (23) insure that d_i, the amount of freight delivered at node i, is nonpositive. The constraint in (24) is required to insure that the total amount of freight that is picked up in a tour is delivered. The constraints in (25) are subtour elimination constraints which are added to the MIP as needed. These constraints eliminate submaximal subtours.

For both the tabu search and MIP, the cluster/routing procedure is applied to each company's node set, then the delivery costs for any contested nodes are calculated, and the company with the lowest cost is awarded the node and the node is removed from the node sets of the losing companies. The process is then repeated with the adjusted node sets until all nodes are serviced at the lowest cost. The solution procedures presented in the previous sections are applied to a series of sample problems next.

RESULTS AND DISCUSSION

A series of 9 sample problems was generated in which the number of competing trucking companies was varied from 2 to 8, the number of nodes from 80 to 150, and the degree of market transparency from 0% to 50%. An example of a problem with 4 companies, 120 nodes, and a degree of market transparency of 30% is shown in Figure 5. These problems were solved using the cluster first, route second approach outlined in Figure 2 with the routing done by tabu search and the MIP formulation described above. Depending on problem size, the GAP and MIP routing solutions were obtained using either XPress-MP, Dash Optimization (2002) or the NEOS server (see Czyzyk, et al., (1998) and Gropp, et al., (1997)).

To compare the performance of these two methods, appropriate parameters must be identified. The most obvious measure of performance is the profit function, equation (9). It seems clear that the key indicator of performance is profit. However, total profit does not take into account any measure of operational efficiency. Two possible measures of efficiency are the number of tours and the number of hours required to service all available nodes. Of these two measures, the number of tours appears to be more problematic. On the one hand, using fewer trucks lowers labor and fuel costs and vehicle wear and tear. On the other hand, more tours for a given number of nodes means shorter tours which tend to result in shorter commodity trips and, in turn, lower travel costs. In addition, using more tours of shorter length puts the company in a better position to meet whatever demand arises in the future in that new stops can be added at lower cost. Moreover, in the event of a vehicle breakdown, a company which was using shorter tours would be in a better position to handle the emergency. Profit per hour would appear to be preferable because it is a more precise measure of efficiency, the total number of hours that it takes to satisfy the freight needs of the nodes available to the company rather than the total number of required tours which could vary considerably in length. To compare the performance of the two methods, the following parameters were calculated:

(1) total profit for game: $prof_{total} = \sum_{i=1}^{np} \sum_{j=1}^{ntour_i} prof_{ij}$ (28)

(2) profit per tour for game: $prof_{tour} = \sum_{i=1}^{np} \left(\frac{\sum_{j=1}^{ntour_i} prof_{ij}}{ntour_i} \right)$ (29)

(3) profit per hour for game: $prof_{hour} = \sum_{i=1}^{np} \left(\frac{\sum_{j=1}^{ntour_i} prof_{ij}}{\sum_{j=1}^{ntour_i} t_{tour\,ij}} \right)$ (30)

(4) game total distance traveled: $dist_{total} = \sum_{i=1}^{np} \sum_{j=1}^{ntour_i} dist_{ij}$ (31)

(5) game total tour time duration: $t_{tour\,total} = \sum_{i=1}^{np} \sum_{j=1}^{ntour_i} t_{tour\,ij}$ (32)

Where np is the number of companies, $ntour_i$ is the number of tours for company i, $prof_{ij}$ is the profit for the jth tour of company i, t_{tourij} is the tour time for the jth tour of company i, and $dist_{ij}$ is the distance traveled in the jth tour of company i.

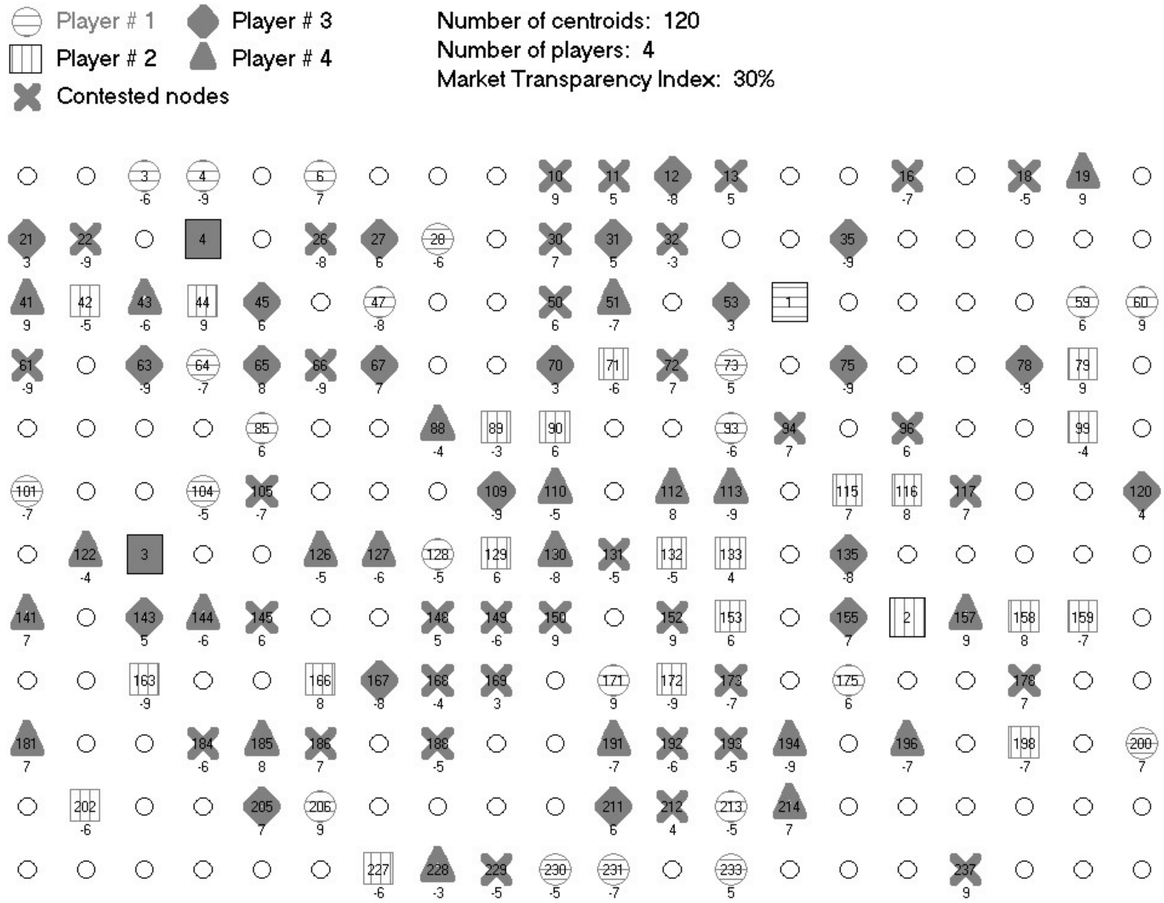


Figure 5: A sample problem

Table 1 compares the parameter values for the tabu and MIP solutions and Table 2 shows the relative differences in these parameters with respect to the MIP values. Comparing the profit values for the tabu search and MIP solutions, in all nine problems, the total profit and profit per tour values were less than 0.70% lower for the tabu search solutions and the profit per hour value was no more than 2% lower than the corresponding MIP value. In all 9 problems, the total tour times were less than 1.5% longer in the tabu search solutions than in the MIP solutions. Similarly, in all 9 problems, the total distance traveled values were less than 4.5% longer in the tabu search solutions than in the MIP solutions. The tabu search solution for the problem in Figure 5 is shown in Figure 6.

While the MIP routing approach obtained better results, it is not as practical as the tabu search approach for a number of reasons. The number of variables and constraints are both $O(N^2)$, so the computational effort becomes very large for larger problems. For small problems ranging from 8 to 14 nodes, the average running time was almost 1.5 seconds with an average of 235 iterations. For the same set of problems, the tabu search required an average of 25 iterations to obtain solutions. Another advantage of the tabu search approach is that it is not restricted to looking at the linear objective of maximizing profit as is the MIP approach. The tabu search could be modified to consider other objectives such as profit per hour which are not linear.

Table 1: Performance measures for the tabu search and MIP solutions

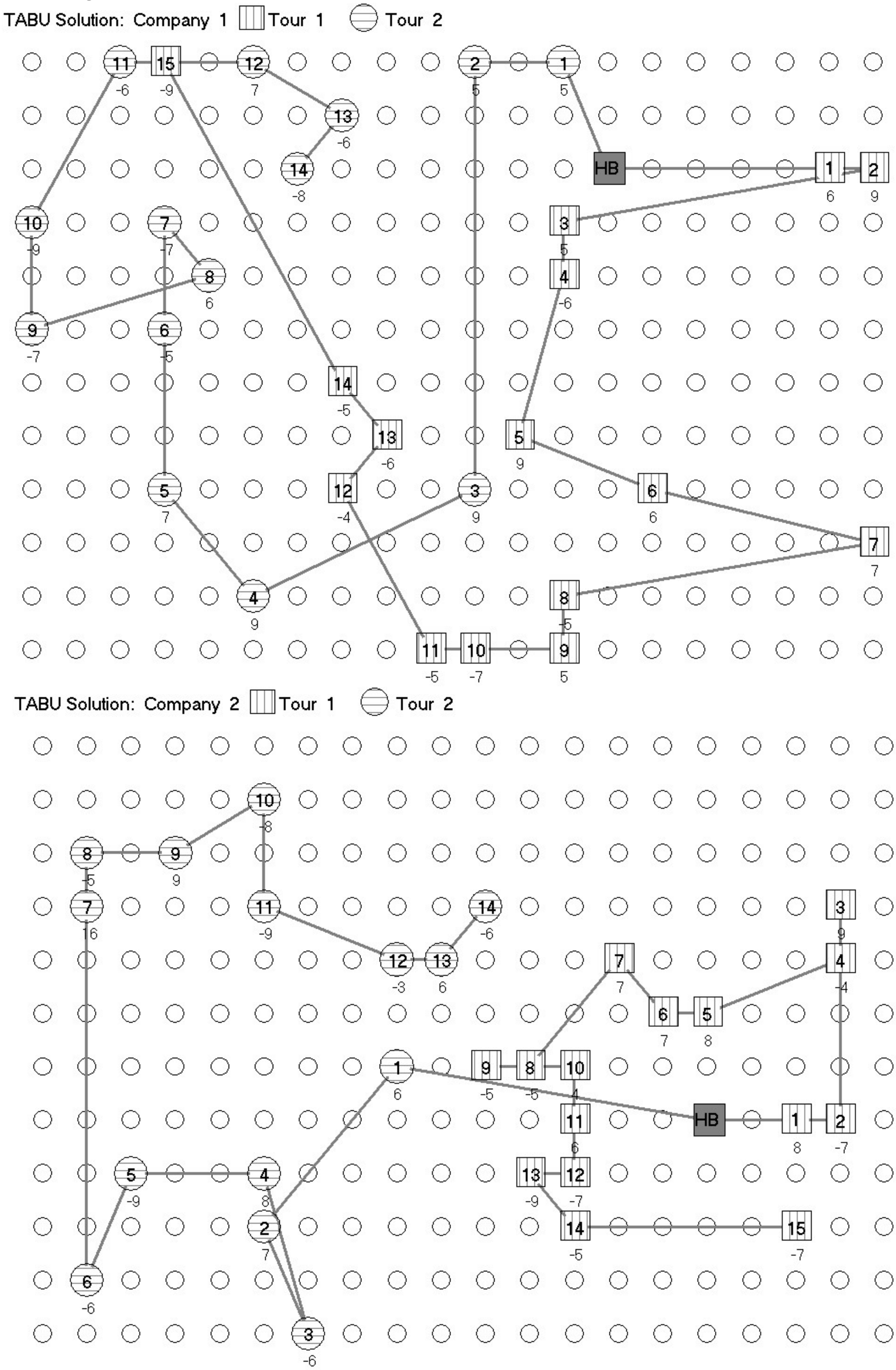
Problem	Solution	Total Profit	Profit/tour	Profit/hour	Tourtime	Total Dist
A (2 carriers, 80 nodes,	MIP	3599.81	1199.94	158.35	45.48	314
0% node overlap)	TABU	3596.08	1198.69	157.82	45.58	316
B (2 carriers, 80 nodes,	MIP	3796.80	1265.60	169.82	44.72	280
10% node overlap)	TABU	3789.32	1263.11	168.73	44.92	284
C (2 carriers, 80 nodes,	MIP	3631.16	1210.39	156.31	46.33	324
20% node overlap)	TABU	3616.21	1205.40	154.34	46.73	332
D (2 carriers, 80 nodes,	MIP	4665.32	1349.63	172.77	54.12	324
25% node overlap)	TABU	4654.11	1346.51	171.35	54.42	330
F (2 carriers, 80 nodes,	MIP	6163.39	1540.85	177.57	69.38	406
30% node overlap)	TABU	6152.17	1538.04	176.49	69.68	412
F' (4 carriers, 120 nodes,	MIP	6073.11	2771.01	329.38	74.13	490
30% node overlap)	TABU	6031.99	2753.57	322.79	75.23	512
G (4 carriers, 150 nodes,	MIP	7591.87	2763.86	339.72	88.83	552
50% node overlap)	TABU	7565.71	2753.89	335.71	89.53	566
H (8 carriers, 150 nodes,	MIP	7564.93	4094.22	560.25	107.90	852
30% node overlap)	TABU	7546.24	4084.88	556.54	108.40	862
I (8 carriers, 150 nodes,	MIP	7472.33	3736.16	552.07	107.08	848
50% node overlap)	TABU	7449.90	3724.95	547.38	107.68	860

Table 2: Differences in tabu search performance measures relative to MIP values (%)

Problem	Solution	Total Profit	Profit/tour	Profit/hour	Tourtime	Total Dist
A (2 carriers, 80 nodes,	MIP	0.00	0.00	0.00	0.00	0.00
0% node overlap)	TABU	-0.10	-0.10	-0.33	0.22	0.64
B (2 carriers, 80 nodes,	MIP	0.00	0.00	0.00	0.00	0.00
10% node overlap)	TABU	-0.20	-0.20	-0.64	0.45	1.43
C (2 carriers, 80 nodes,	MIP	0.00	0.00	0.00	0.00	0.00
20% node overlap)	TABU	-0.41	-0.41	-1.26	0.86	2.47
D (2 carriers, 80 nodes,	MIP	0.00	0.00	0.00	0.00	0.00
25% node overlap)	TABU	-0.24	-0.23	-0.82	0.55	1.85
F (2 carriers, 80 nodes,	MIP	0.00	0.00	0.00	0.00	0.00
30% node overlap)	TABU	-0.18	-0.18	-0.61	0.43	1.48
F' (4 carriers, 120 nodes,	MIP	0.00	0.00	0.00	0.00	0.00
30% node overlap)	TABU	-0.68	-0.63	-2.00	1.48	4.49
G (4 carriers, 150 nodes,	MIP	0.00	0.00	0.00	0.00	0.00
50% node overlap)	TABU	-0.34	-0.36	-1.18	0.79	2.54
H (8 carriers, 150 nodes,	MIP	0.00	0.00	0.00	0.00	0.00
30% node overlap)	TABU	-0.25	-0.23	-0.66	0.46	1.17
I (8 carriers, 150 nodes,	MIP	0.00	0.00	0.00	0.00	0.00
50% node overlap)	TABU	-0.30	-0.30	-0.85	0.56	1.42

a)

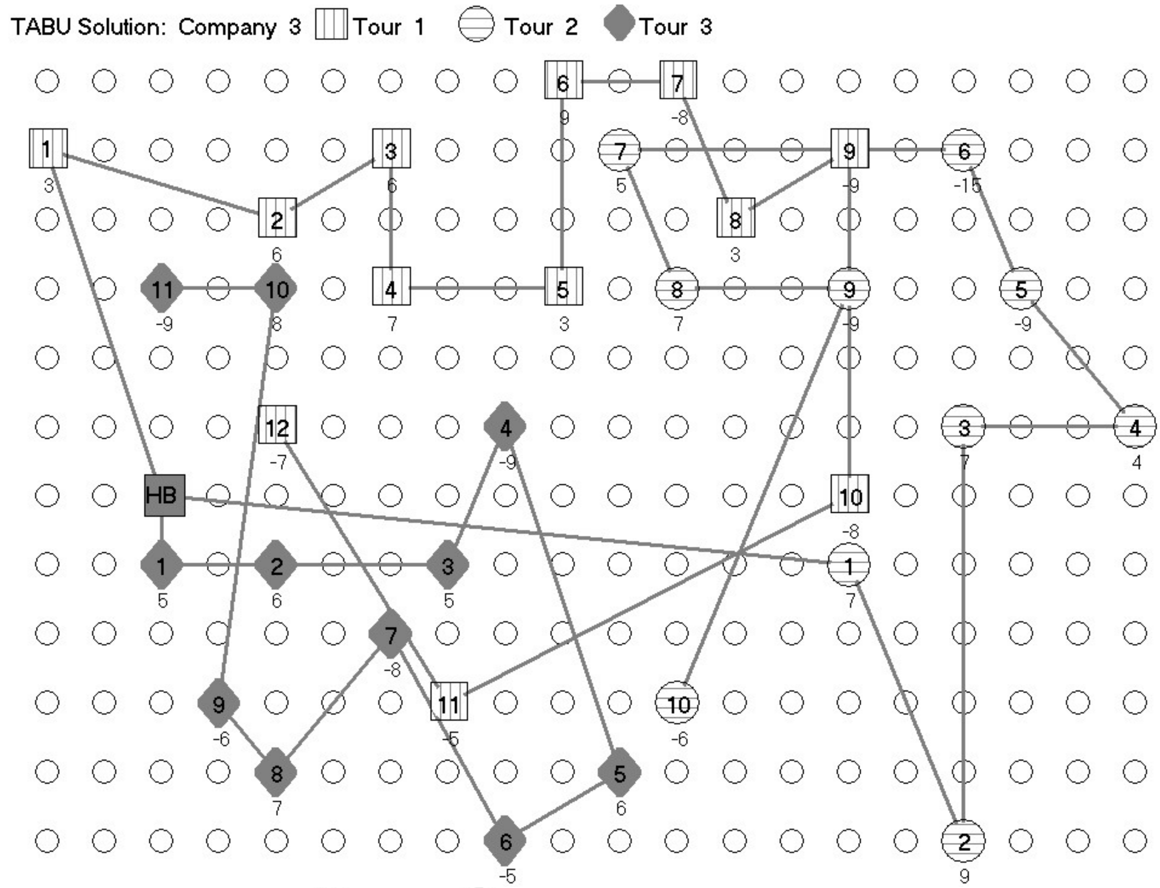
b)



Note: The stops in each tour are numbered sequentially, HB indicates the company's home base, and the numbers below each stop indicate the node's production (> 0) or the attraction (< 0).

Figure 6: The tours for problem F`(4 companies, 120 nodes, and market transparency index of 30%) for: (a) company 1 and (b) company 2

(c)



(d)

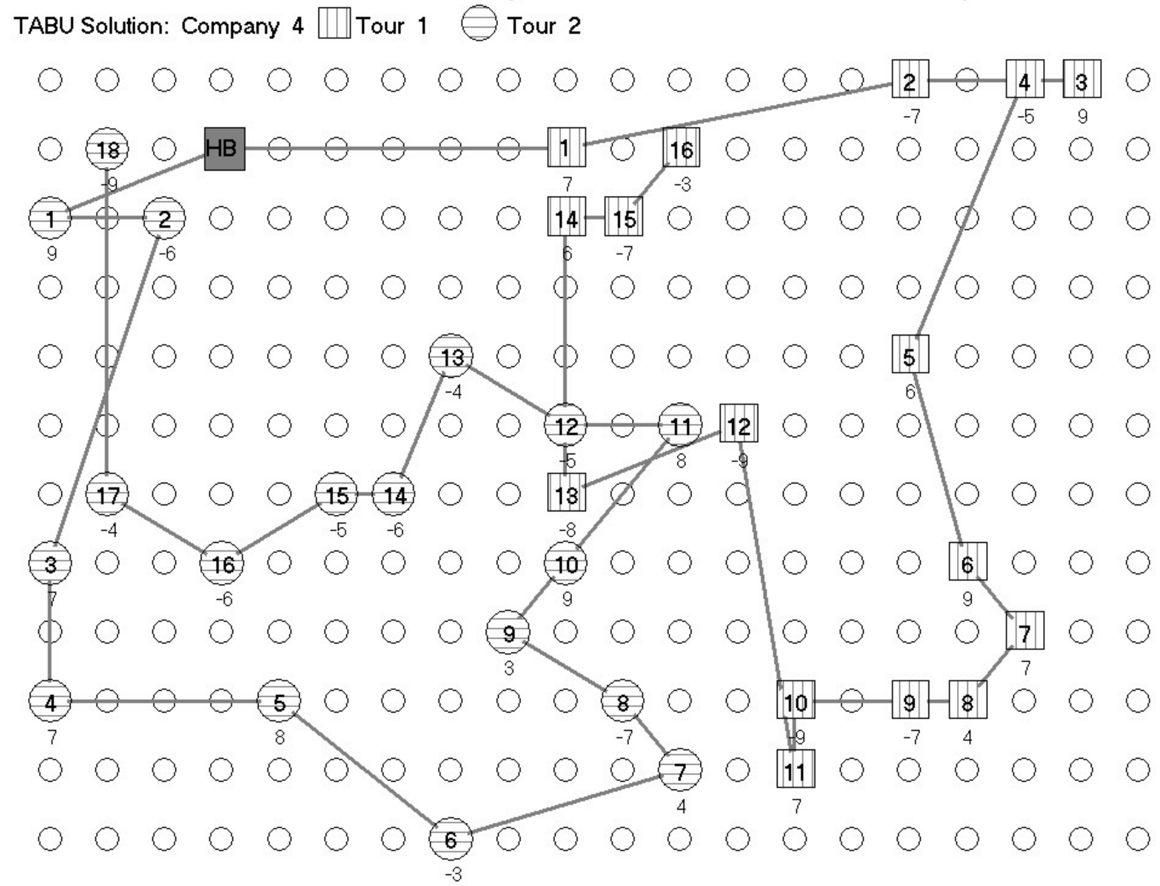


Figure 6 continued: The tours for problem F` for: (c) company 3 and (d) company 4

As discussed before, to analyze the role of market transparency, a specified percentage of nodes are available to more than one company. The percentage is designated as ρ and, as it increases, the transparency and competitiveness of the market increases. It can be hypothesized that, as the market transparency increases with increasing ρ, the market will become more segmented geographically, that is, the nodes will tend to be won by the player whose home base they are closest to. To test this hypothesis, the node overlap in game G (four players, 50% node overlap) was reduced to 25% and increased to 75% and 100%. Tabu search solutions were obtained for each value of ρ and the distance between each node and the home base of the company which won the node was calculated. If the hypothesis that increasing market transparency by increasing ρ results in an increase in market segmentation is true, then the node-home distance should decrease as ρ increases. In each case except for ρ = 100%, there are a number of nodes which are available to only one player and, regardless of where they are located, that player must service them. Thus, to fairly test the hypothesis, the analysis should focus only on the contested nodes, that is, the nodes which are available to more than one company. This is because the nodes that only have one carrier serving them are in no position to switch to another company.

Figure 7 shows the contested node-home base distance cumulative frequency distribution in each case. This figure lends support to the hypothesis. The 100% case has the highest frequency in the four lowest distance intervals. It has the highest frequency in five out of the eight lowest distance intervals, while the 25% case generally has the lowest frequency for these intervals. The frequency distributions were modeled using the following gamma function:

$$freq = a_0 e^{a_1 dist} dist^{a_2} \tag{33}$$

Figure 8 shows the frequency distributions and the estimated gamma functions when the noncontested nodes are excluded. The mode for these modeled distributions tended to shift to the right as ρ decreases with the mode for 100% at 0.30 miles, the mode for 75% at 0.40, and the mode for 50% at 0.35 miles, and the mode for 25% at 0.75 miles. Table 3 shows the totals and means for the node- home base distance. The mean for this distance generally decreased as ρ increases.

Table 3: Distance between contested nodes and home base of company which won them

rho	Contested Nodes	Home base-node distance	
		Total	Mean
25%	37	21.50	0.5810
50%	73	37.90	0.5192
75%	104	56.75	0.5457
100%	145	75.00	0.5172

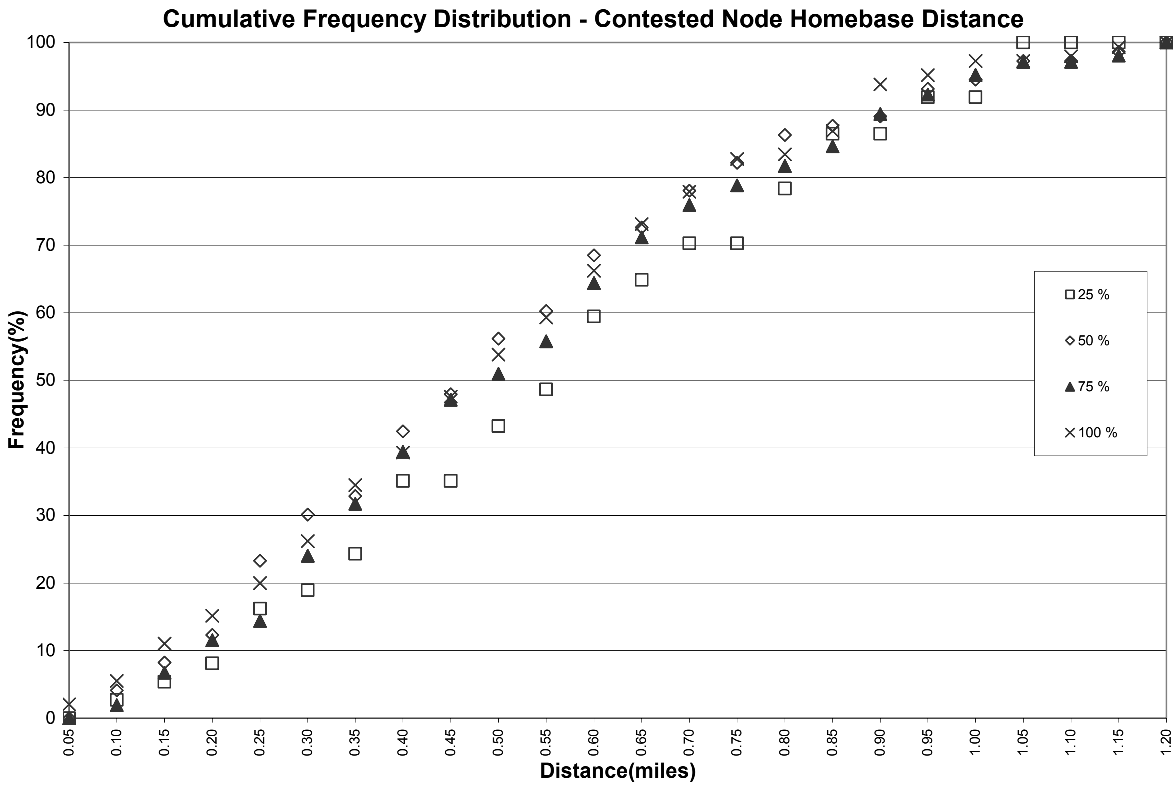


Figure 7: Cumulative frequency distributions of contested node-home base distances

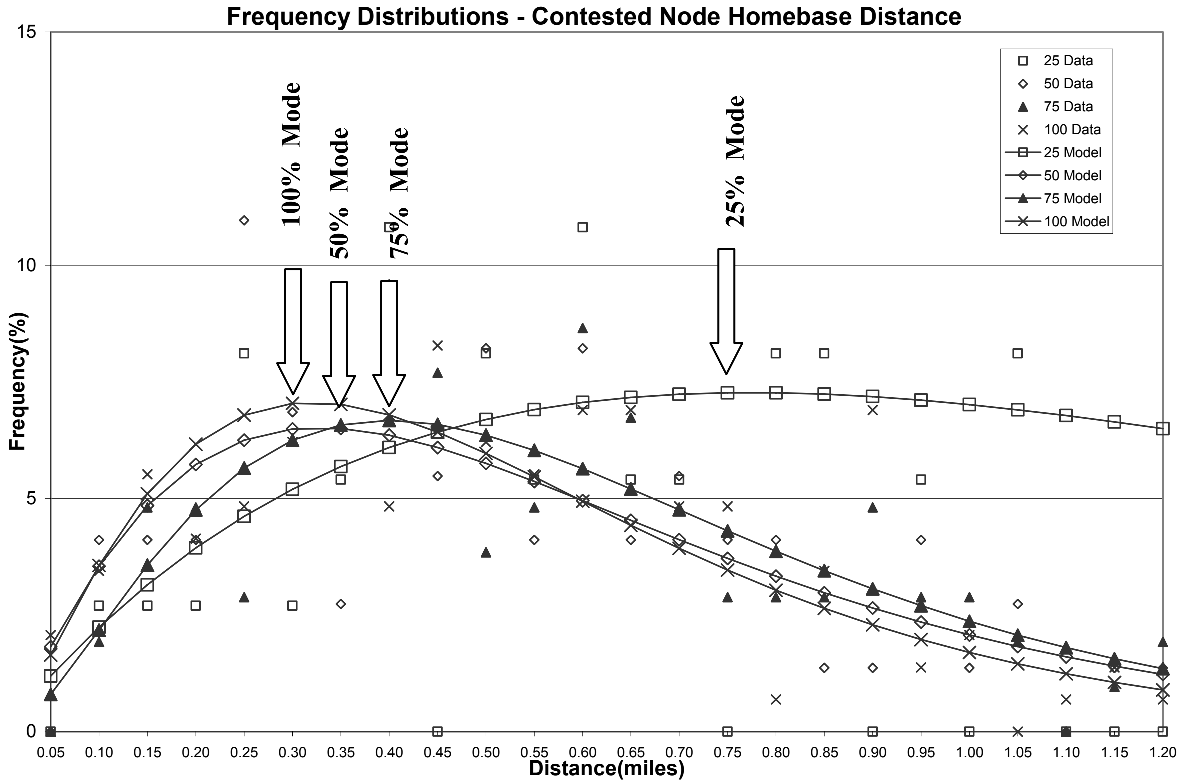


Figure 8: Modeled frequency distributions of contested node-home base distances

CONCLUSION

This paper presents a solution method for the multiple vehicle routing problem with profits and competition (MVRPPC). The method uses a "cluster first, route second" approach in which the first stage consists of a geometric clustering combined with a generalized assignment problem (GAP) to create clusters from which feasible tours can be constructed and the routing stage is performed using a tabu search. For comparison purposes, the routing was also done using a mixed integer programming (MIP) formulation.

Competition among the trucking companies was introduced into the process by the fact that a specified proportion of the nodes were available to more than one company. For these overlapping nodes, the transportation costs for each company servicing the node were then calculated and these nodes were removed from the node sets of players who did not have the lowest cost and the clustering/routing procedure was repeated with the new node sets.

This process was continued until each node was included in exactly one tour at lowest cost resulting in an approximation of a modified spatial price equilibrium.

This solution method was applied to a series of sample problems of varying size and complexity. A comparison of the tabu search and MIP solutions indicated that the tabu search solutions were only slightly less profitable than the corresponding MIP solutions. In all nine problems, total profit and profit per tour values were less than 0.70% lower for the tabu search solutions and the profit per hour value was no more than 2% lower than the corresponding MIP value. These results indicate that the clustering/GAP/tabu search solution approach presented here appears to be a flexible, efficient method for solving the MVRPPC which is quite accurate as indicated by the comparison to the MIP solutions. The tabu search approach has several advantages over the MIP approach including requiring considerably fewer iterations to obtain solutions and the capability of considering non-linear objectives.

The paper analyzed the effect of varying the degree of market transparency to test the hypothesis that increasing the transparency of the market will increase the geographic segmentation of the market. Evidence in support of this hypothesis was presented and it was concluded that, there appeared to be a trend of increasing market segmentation with increasing market transparency.

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