Increasing the Resiliency of Local Supply Chain Distribution Networks against Multiple Hazards

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Abstract

We examine the resiliency of retail locations of a supply chain network to aid in the recovery of the local community after an extreme event. A two-stage stochastic programming model is used to determine the placement of permanent generators at the retail locations of Stewart's Shops, which distributes both convenience items and fuel in Upstate New York and Vermont, to enhance the resiliency of the supply chain. Our measure of resiliency specifically considers the recovery process of each retail location after the extreme event and its interdependency on other external infrastructure systems. Our computational experiments consider the multiple distinct types of hazards that can affect the retail locations of Stewart's Shops. We empirically explore different stochastic sampling procedures to solve the resiliency model. The results of computational tests indicate that we can converge to an approximate optimal solution quickly. We compare the resiliency efforts when planning for different types of hazards versus all hazards simultaneously as well as the impact of external infrastructure systems on the resiliency efforts. The empirical study identifies that the stores in rural, less densely populated areas, serving a large population should be selected to receive generators.

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1 Introduction

Recent extreme events, such as Hurricanes Irene in 2011 and Sandy in 2012 that affected New York and New Jersey, have demonstrated the need for enhancing the resiliency of supply chain systems. This is especially important for local supply chain networks that move critical goods, such as food, batteries, and fuel, into the areas affected by the extreme event. These critical goods allow the *local population* to begin to recover from the event and, often, companies operate hybrid retail operations that are part convenience stores (to provide food and batteries) and part gas station (to provide fuel). As a motivating example, Stewart's Shops is a company that operates 330 convenience stores and gas stations locations in Upstate New York and Vermont. Figure 1 presents a map of the retail locations of this company relative to the northeastern United States. Figures in the remained of this work will zoom in to the black box for clarity. Stewart's has 58 locations that operate just as convenience stores and 272 locations that operate as both convenience stores and gas stations (see Stewart's Shops [23]). In the past few years, multiple types of hazards have impacted Stewart's retail locations including hurricanes (in particular, Hurricanes Irene and Sandy), flooding, blizzards, and ice storms. This means that any efforts to increase the resiliency of Stewart's Shops for delivering critical goods to local populations should incorporate the potential impact of these various hazards.

The resiliency of a local supply chain distribution network, like that of Stewart's Shops, is typically focused on its ability to bounce back from disruptions. For local distribution networks, an important aspect of its bounce back is the capability to have its retail operations open for business. There are both *internal* and *external* factors that determine when a retail operation can begin its vital role as a distribution point after the disruptive event. The internal factors typically involve the steps necessary to reopen the store after any damage that was caused by the event. Example internal factors include: (i) clearing debris, snow, or ice from the parking and refueling areas, (ii) cleaning up the interior of the store and restocking shelves, (iii) rebooting information systems, and (iv) having workers arrive at the store, which often times depends on external transportation systems. The external factors involve whether the services (such as power and telecommunications) necessary to support

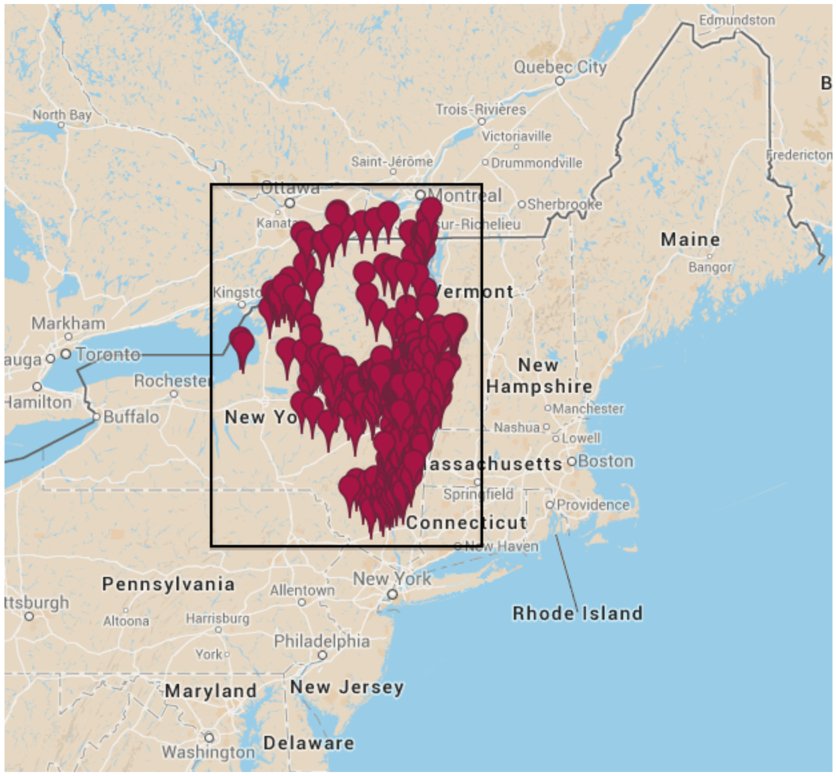


Figure 1: Locations of Stewart's Shops.

the retail operations are available after the event. For example, after Hurricane Sandy, lack of electrical power was a major source of the delay in the re-opening of gas stations in the New York/New Jersey areas (see, for example Ma [10], Zernike [24], Lipton and Krauss [8], Hu and Yee [4], and Goldberg [2]). In fact, generators were brought in to the area by certain gasoline companies for the sole purpose of reopening their points of distribution (Goldberg [2]). In addition, many gas stations had their telecommunications services disrupted by Hurricane Sandy implying that they were only able to accept cash from customers (for example, Hu and Yee [4]). Therefore, our proposed resiliency models will specifically incorporate the reliance of reopening convenience and fuel distribution on the services provided by other (potentially disrupted) infrastructure systems.

The focus of this work is on locating *permanent* generators at the retail operations of a local supply chain distribution network in order to increase the resiliency of the system against multiple types of hazards. The resiliency of a particular retail operation is measured as the sum of the weighted (by demand) opening time for various commodities (e.g., cashpaying customers for fuel). The calculation of the opening time of a commodity at a retail operation will incorporate both the internal and external factors (e.g., dependencies on power and telecommunications) that affect it. This means that we use a two-stage stochastic pre-planning model where the first stage decisions locate the generators and the second stage, for each scenario, captures the resiliency of the distribution network for a realization of the damage of a hazard.

We do note that the 'opening times' will not need to incorporate the arrival of inventory into the retail operation - the types of items that tend to be sold immediately after a disruptive weather event will be well-stocked prior to that event and are often able to be replenished shortly after the event. Therefore, the focus of the resiliency measure should be on the ability to distribute on-site inventory rather than receiving shipments from elsewhere. It should be noted that Stewart's Shops will send out more 'necessity' items to stores in areas potentially affected by an incoming event. In addition, the lack of power at gas stations was a major concern with respect to the gasoline shortages and rationing after Hurricane Sandy. Therefore, the particular focus on opening times of retail operations is well-justified. This implies that we do not necessarily need to model the underlying warehouse-retailer distribution network in each scenario, thus allowing us to develop fast algorithms to solve our resulting resiliency models.

Our work is related to research on planning for supply chain disruptions; Snyder et al. [22] and Snyder [21] provide an overview of design and fortification models in this mean. These two-stage stochastic models they describe consider the location of supply chain components as the first stage decisions and customer assignment as the second stage decisions. The model in this paper is distinct from this work because our first stage changes the properties of existing components and our second stage focuses on the time to recovery of distribution points.

There has been previous research on two-stage pre-planning models for locating emergency supplies before a disaster. These models incorporate the dependencies of the supply chain on the transportation network by having travel times be scenario-dependent based on the damage of the event - see, for example, Shen et al. [19], Van Hentenryck et al. [3], Mete and Zabinsky [11], Rawls and Turnquist [15], and Salmeron and Apte [16]. However, these models *do not* consider the reliance of the supply chain on other critical civil infrastructure systems, such as power and telecommunications. Shen [18] examines building in new arcs in a network to increase the resiliency of interdependent infrastructure systems; however, these models assume that the infrastructures will 'work together' in the second stage in terms of planning their recovery efforts from the event. This is often not the case for infrastructure restoration and, more importantly, local supply chain distribution networks will tend not to have a voice in restoration efforts from large-scale events. Further, these previous works on pre-planning models tend to focus on scenarios that are generated from a single type of event (such as an earthquake Dodo et al. [1] and Liu et al. [9]) rather than multiple hazards.

Other two-stage pre-planning models examine the resilience of infrastructure systems subject to extreme events. Liu et al. [9] examine the retrofitting of transportation networks; Miller-Hooks et al. [12] examine a freight transportation network and determine the optimal allocation of preparedness and recovery actions; and Peeta et al. [14] consider a highway network and seek to maximize the connectivity after the event. All of these studies do not incorporate the impact of external factors, such as the impact of power outages, on the recovery or resilience of the transportation network. The time it takes to bounce back to normal after an extreme event is one dimension of resiliency which is often ignored in these previous works that is specifically included in our model. A notable exception is Sheu [20] who has examined the time until relief is distributed after a disaster. However, Sheu [20] focuses on a multi-objective model where the supply chain is not explicitly impacted but instead the demand is dynamic based on the degree of impact to different geographical locations.

Main Contributions The main contributions of this work include (i) the consideration of multiple types of hazards in supply chain resiliency planning, (ii) development of a two-stage stochastic program to enhance the resiliency of local supply chain networks specifically considering the recovery process of each retail operations and its interdependency on external infrastructures, and (iii) an empirical exploration of stochastic sampling procedures to solve resiliency models.

The work proceeds as follows: Section 2 introduces the mathematical model and associated algorithm used to solve the two-stage stochastic supply chain resiliency problem; Section 3 presents the results of the computational analysis including the inclusion of different distribution of hazards and the impact of internal and external factors; we conclude in Section 4. Please see the Appendix for corresponding parameter and data generation for each hazard scenario.

2 Mathematical Model and Algorithms

The proposed mathematical model to increase the resiliency of local supply chain distribution networks involves locating generators at retail operations to minimize the weighted 'opening time' of the retail operations across a set of scenarios. The *opening time* of the retail operation for commodity ℓ at store j requires that all internal and external factors that could prevent the opening are complete. In particular, the internal factors include: (i) completing all work (such as repairing damage) for tasks that do not necessarily require power (we will refer to these as 'non-power tasks') and (ii) completing all work on tasks that do require power (we will refer to these as 'power-based tasks'). Note that there may be non-power tasks (such as cleaning the store) that can be completed faster if power is available at the store - this will be incorporated in our model. The external factors include: (i) any (potential) flooding around the store subsiding, (ii) power being restored to the store (if there is no generator) and (iii) telecommunications being restored to the store (for credit-only customers).

To provide a formal description of the model, it is necessary to provide an overview of the notation used to describe various parameters associated with the problem. The relevant parameters include:

- S is the set of all scenarios.
- N is the set of all stores.
- L is the set of all commodities.
- w_s is the weight of scenario $s \in S$. This would typically be viewed as the probability of the event occurring; however, for our purposes, this weight reflects the fact that

we care about multiple types of hazards. A scenario for a type of hazard would have a probability and the importance of that hazard multiplied by this probability would give us the weight for the scenario.

- $d_{j\ell}$ is the cash demand for store j and commodity ℓ.
- $\bar{d}_{j\ell}$ is the credit-only demand for store j and commodity ℓ.
- K is the number of generators available.
- r^p_{sj} is the release time (or restoration time) for power (from the grid) at store j in scenario s.
- r^c_{sj} is the release time for communications at store j in scenario s.
- r^f_{sj} is the release time for any floodwaters at store j in scenario s. We assume that $r^f_{sj} \leq r^p_{sj}$ for all s and j since power will not be restored to areas damaged by flooding until the flooding subsides and proper electrical inspections are done (see, for example, Issler and Brodsky [5] for discussion of this issue after Hurricane Sandy).
- $p_{sj\ell}$ is the time needed to complete power-based tasks at store j for commodity ℓ and scenario s.
- $w_{sj\ell}$ is the work needed to be completed at store j for commodity ℓ and scenario s for non-power tasks.
- σ^p_j is the speed the work associated with non-power tasks is completed when power is available.
- σ^{np}_j is the speed the work associated with non-power tasks is completed when power is not available.

The mathematical model is a two-stage stochastic program where the first stage decisions locate generators at the retail operations of the local distribution supply chain network and the second stage decisions calculate the opening times of each store and commodity in each scenario. To this end, we define binary decision variables z_j for $j \in N$ that represent the decision of locating a generator at store j. The decision variables $C_{sj\ell}$ and $\bar{C}_{sj\ell}$ provide the opening time of commodity ℓ at store j in scenario s for cash customers and credit-only customers, respectively. The mathematical model of our resiliency model for local supply chain distribution networks (R-LSC) is then:

$$\min_{z,C,\bar{C}} \sum_{s \in S} w_s \sum_{j \in N} \sum_{\ell \in L} d_{j\ell} C_{sj\ell} + \bar{d}_{j\ell} \bar{C}_{sj\ell} \tag{1}$$

subject to: (R-LSC)

$$\sum_{j \in N} z_j \leq K \tag{2}$$

$$C_{sj\ell} \geq r^p_{sj}(1 - z_j) + p_{sj\ell} \qquad \forall s \in S, \forall j \in N, \forall \ell \in L \tag{3}$$

$$C_{sj\ell} \geq \left(r^p_{sj} + \frac{w_{sj\ell} - (r^p_{sj} - r^f_{sj})\sigma^{np}_j}{\sigma^p_j} \right)(1 - z_j) + p_{sj\ell} \qquad \forall s \in S, \forall j \in N, \forall \ell \in L \tag{4}$$

$$C_{sj\ell} \geq \left(r^f_{sj} + \frac{w_{sj\ell}}{\sigma^p_j} \right) z_j + p_{sj\ell} \qquad \forall s \in S, \forall j \in N, \forall \ell \in L \tag{5}$$

$$\bar{C}_{sj\ell} \geq C_{sj\ell} \qquad \forall s \in S, \forall j \in N, \forall \ell \in L \tag{6}$$

$$\bar{C}_{sj\ell} \geq r^c_{sj} \qquad \forall s \in S, \forall j \in N, \forall \ell \in L \tag{7}$$

$$z_j \in \{0, 1\} \qquad \forall j \in N. \tag{8}$$

Constraint (2) limits the number of generators placed at the stores while constraints (3) - (7) help to ensure the opening times consider all internal and external factors. In particular, for cash customers, constraints (3) - (5) ensure that the following steps need to be complete prior to opening store j for commodity ℓ: (i) flooding at the store subsides, (ii) all non-power tasks are completed, (iii) power returns (either through a generator or being restored) to the store, and (iv) all power-based tasks are completed. If $z_j = 0$, then constraints (3) imply that we do not begin the power-based tasks until *at least* power is restored to the store and constraints (4) imply that we do not begin the power-based tasks until *at least* the non-power tasks are complete. The first term inside the parenthesis in constraints (4) is the time when we begin processing non-power tasks at their 'power' speed while the second term provides the amount of time required to finish the remaining work on these tasks. Note that if the tasks can be completed before the power speed kicks on, then the second term will be negative, thus constraints (3) will be active. Constraints (6) and (7) ensure the opening time for commodity ℓ of credit-only customers is based on the fact that the store is open for cash customers for commodity ℓ and communications is restored to the store.

It can be expected that the number of scenarios in R-LSC will be extremely large and, therefore, it may not be computationally feasible to solve the form of R-LSC that incorporates all scenarios. Therefore, we will apply sample average approximation (see Shapiro et al. [17]) to determine an (approximate) optimal solution to R-LSC. For our case study of applying R-LSC to locating generators at Stewart's Shops, the set of scenarios span four distinct types of hazards: hurricanes, flooding, blizzards, and ice storms. The weight of a particular scenario, w_s, will be based on (i) the priority of the type of hazard associated with scenario s and (ii) the probability that this type of hazard produces scenario s. For example, if we care about all four hazards equally, and a particular hurricane scenario has a .5 probability of occurring and a particular flooding scenario has a .2 probability of occurring, then the weights will be selected as .5 and .2 for these scenarios, respectively.

Our computational analysis will first apply sample average approximation (SAA) techniques to R-LSC to locate generators to increase the resiliency against a single type of hazard. The purpose of this is twofold: first, it will help determine the limits of planning for a particular type of hazard versus planning for all hazards and secondly, it will be the basis for an empirical sampling mechanism when protecting against multiple types of hazards. We will then explore different ways to sample when considering the multiple types of hazards and see their impact on the convergence of the SAA.

It turns out that R-LSC can be solved in $O(|N||S||L|)$ time. The method to solve R-LSC comes from the observation that the second-stage decisions are decomposable by store and that the generator location decisions can be viewed as *improving* the worst case opening times for each scenario. In particular, we define R_{sj} to be the 'base' resiliency measure, i.e., the resiliency of store j in scenario s when a generator is not located at j:

$$R_{sj} = \sum_{\ell \in L} d_{j\ell} \left(\max \left\{ r^p_{sj}, r^p_{sj} + \frac{w_{sj\ell} - (r^p_{sj} - r^f_{sj})\sigma^{np}_j}{\sigma^p_j} \right\} + p_{sj\ell} \right)$$

$$+\bar{d}_{j\ell}\left(\max\left\{\max\left\{r^p_{sj}, r^p_{sj} + \frac{w_{sj\ell} - (r^p_{sj} - r^f_{sj})\sigma^{np}_j}{\sigma^p_j}\right\} + p_{sj\ell}, r^c_{sj}\right\}\right)$$

This calculation simply forces constraints (3)-(4) and (6)-(7) to be 'active' for scenario s and store j. We then define R'_{sj} to be the objective for scenario s at store j when a generator is located there:

$$R'_{sj} = \sum_{\ell\in L} d_{j\ell}\left(r^f_{sj} + \frac{w_{sj\ell}}{\sigma^p_j} + p_{sj\ell}\right) + \bar{d}_{j\ell}\left(\max\left\{r^f_{sj} + \frac{w_{sj\ell}}{\sigma^p_j} + p_{sj\ell}, r^c_{sj}\right\}\right).$$

Note that $R_{sj} \geq R'_{sj}$ for all s and j because $r^f_{sj} \leq r^p_{sj}$. We can then reformulate R-LSC as

$$\min \sum_{s\in S} w_s \sum_{j\in N} R_{sj}(1 - z_j) + R'_{sj}z_j = \sum_{j\in N}\sum_{s\in S} w_s R_{sj} + \sum_{j\in N}\sum_{s\in S} w_s(R'_{sj} - R_{sj})z_j \tag{9}$$

subject to: (KP)

$$\sum_{j\in N} z_j \leq K \tag{10}$$

$$z_j \in \{0,1\} \qquad \forall j \in N. \tag{11}$$

This reformulation is a knapsack problem where all items (stores) have unit coefficients in the knapsack constraint. Therefore, the K stores with the lowest values of $\sum_{s\in S} w_s(R'_{sj} - R_{sj})$ will be chosen to have generators located at them. It requires $O(|S||L|)$ time to calculate R_{sj} and R'_{sj} for each store and, therefore, R-LSC can be solved in $O(|N||S||L|)$.

3 Computational Analysis

The purpose of this section is to explore both empirical convergence properties of R-LSC and to provide policy-based analysis for Stewart's Shops. The area which Stewart's Shops operate their retail stores is prone to four types of hazards: hurricanes, flooding, blizzards, and ice storms. Each of these hazards has their own unique properties in terms of how they come into the area and what damage they tend to cause to the retail stores, power grid, and telecommunications infrastructure. The Appendix discusses our techniques to generate a scenario for each of these distinct types of hazards.

For our case study, we consider two types of commodities: the 'convenience' (e.g. food, water, and batteries) commodity and the 'fuel' commodity. The demand level for these commodities at store $j \in N$ are a function of the location of the store, its surrounding areas, and its capabilities. We first determine the overall demand level for commodity $\ell = \{c, f\}$ (where $c =$ convenience and $f =$ fuel) through the following procedure: (1) for each county and commodity ℓ, determine the set of Stewart's Shops in that county capable of delivering that commodity and (2) split the demand (which we measure as the population, see U.S. Census [13]) of the county evenly among all stores capable of delivering that commodity. We then assume that 50% of this overall demand for store j and commodity ℓ is 'credit-only' to determine $d_{j\ell}$ and $\bar{d}_{j\ell}$. As an example, if there is a county with 10,000 residents and 10 Stewart's Shops that have gas stations, then we assume that the overall demand for fuel at each of these locations is $1,000$. Figure 2 displays a heat map of the Stewart's stores based on their total level of demand over all types of customers (cash and credit only) and commodities (convenience and fuel).

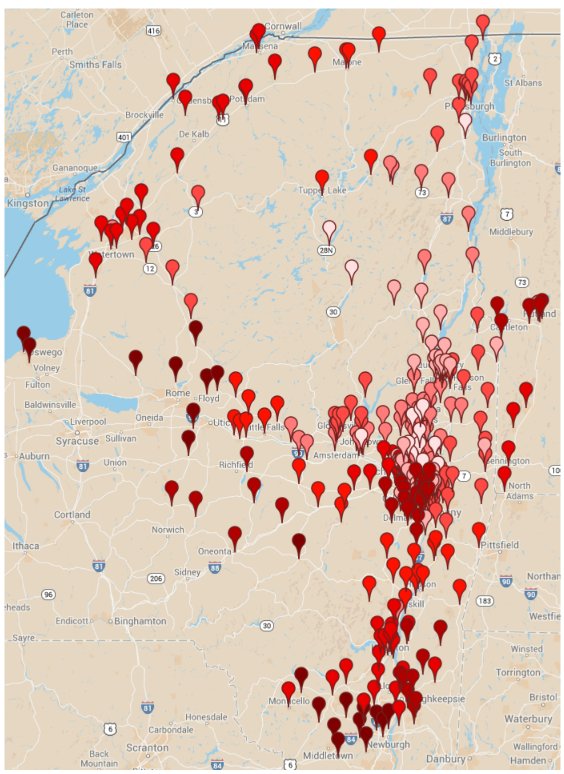


Figure 2: Heat map displaying the 330 Stewart's Stores and their respective total demand level, where darker shading indicates a higher level of demand.

There are a few limitations to creating our demand levels in this manner. First, it assumes that the entire population will visit a Stewarts shop after an event. However, this assumption is not too limiting since if Stewart's market share is uniform across all counties, the optimal solution will not change since all solutions will have their objective function multiplied by their percentage market share. The second assumption is that it does not factor in the 'closest' Stewart's Shop to a given population - there may be a store across the street that is in a different county and, therefore, the demand is assigned elsewhere. However, the number of these situations across all 330 stores is probably quite small. Further, the assignment of this demand assumes that the population will only visit one Stewart's Shop and will not visit another Stewart's Shop if their shop is closed due to the event, since we assume static and not scenario-based demand. This assumption should be relaxed in future work, however, we justify the assumption by concluding a 'non Stewart's Shop' will often be closer to a closed Stewart's Shop than another Stewart's location and, therefore, the customers for the closed store will visit the 'non Stewart's Shop'.

3.1 Sample Average Approximation for Single Hazard Resiliency

Our first set of tests seeks to determine the location of generators among the 330 Stewart's Shops when considering each type of hazard individually. Specifically, we determine 12 solutions, one for each of the four hazards and three generator levels, $K = 17, 33$, and 50 which represent locating generators at $5\%, 10\%$, and 15% of the number of stores, respectively. As with any empirical stochastic programming problem with a large number of scenarios, we must determine the appropriate number of scenarios to consider that provides a close approximation of the true optimal objective function value and solution. Our approach and stopping criteria uses a combination of the implementations presented by Linderoth et al. [7] and Kleywegt et al. [6].

For a fixed number of scenarios $|S|$, we solve M instances of the problem over $|S|$ randomly generated scenarios and determine their associated solutions $z_1, \ldots, z_M$ and objective function values $v_1, \ldots, v_M$. We then solve one instance of the problem by including the union of all $|S|$ scenarios over the M iterations for a total of $M|S|$ scenarios. Denote the solution and objective function value of this instance as z_{M+1} and v_{M+1}. We then calculate the optimality gaps of $v_1, \ldots, v_M$ relative to v_{M+1}. If all of the optimality gaps are within $\pm 1\%$, we stop, otherwise we increment the number of scenarios considered. The specifics of the algorithm are outlined in Algorithm 1.

Algorithm 1 Convergence Stopping Criteria

Set boolean variable *stopping_criteria* equal to 0
Set number of scenarios, $|S| = 500$
Input: Number of generators, K
Input: Number of iterations, M
while *stopping_criteria* $= 0$ **do**
    for $i = 1 : M$ **do**
        Generate $|S|$ independent scenarios
        Solve for objective function value v_i and solution z_i using the $|S|$ scenarios
    end for
    Consider all $M|S|$ scenarios generated
    Solve for objective function value v_{M+1} and solution z_{M+1} using the $M|S|$ scenarios
    Calculate optimality gaps, $\frac{v_{M+1} - v_i}{v_{M+1}}$, for each $i = 1 : M$
    if All optimality gaps are within $\pm 1\%$ **then**
        Set *stopping_criteria* equal to 1
    else
        Set number of scenarios, $|S| = |S| + 500$
    end if
end while
Return v_{M+1} and z_{M+1}

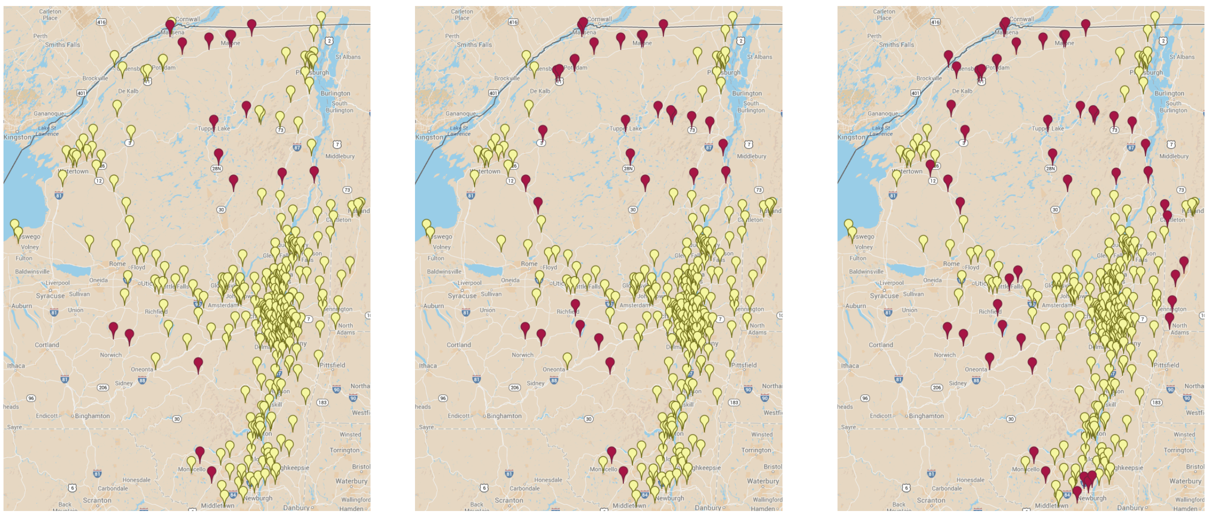
Because we are considering each type of hazard individually, each scenario has the same weight, specifically $\frac{1}{|S|}$. Table 1 displays the results of these tests by showing the types of hazards, the 3 numbers of generators considered, the resulting number of scenarios needed to converge, and objective function value. For reference, we include the objective function value when no generators are placed. From the table, we see that the flooding hazard requires the greatest number of scenarios to converge. This is due to the fact that only 75 stores can ever

Hazard	Number of Generators (K)	Number of Scenarios for Convergence	Objective Function Value
Hurricane	0	-	12,042,400
	17	10,500	11,241,600
	33	11,500	10,876,600
	50	10,000	10,550,900
Flood	0	-	8,129,770
	17	37,500	7,284,080
	33	34,000	6,936,210
	50	29,000	6,886,730
Blizzard	0	-	9,543,590
	17	3,500	8,629,480
	33	3,500	8,237,490
	50	5,500	7,873,700
Ice Storm	0	-	12,696,300
	17	7,000	11,270,400
	33	5,500	10,473,200
	50	8,000	9,784,940

Table 1: Single Hazard Computational Results

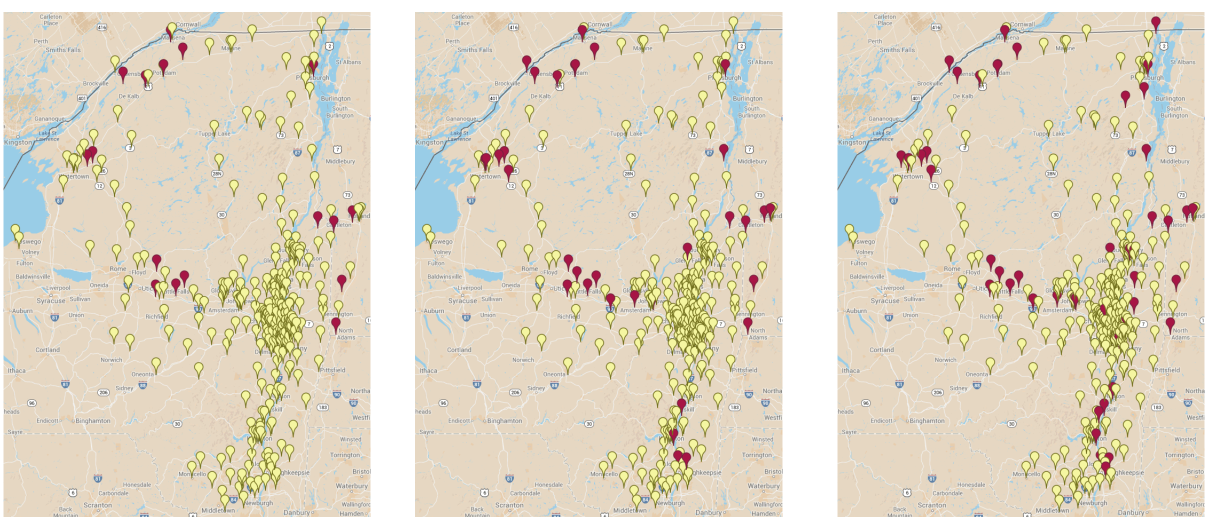
be impacted by a flood, and further over two thirds of the stores are only impacted by at most one particular flooding event. This means that the intersection of stores impacted by two distinct flooding events is small, making it hard to decide on where to locate generators unless many scenarios are considered. Also from the table, we note that the hurricanes and ice storms have the biggest impact for re-opening locations as is realized through the higher objective function values.

Figures 3, 4, 5, and 6 display the solutions for the three generator levels and their associated hazard where darker markers indicate the Stewart's Shops selected to receive a generator. For all four types of hazards, the solutions are incremental as we increase the number of available generators. In other words, the solution to the problem with 17 generators is a subset of the solution with 33 generators, and the solution to the problem with 33 generators is a subset of the solution with 50 generators. This is expected for a fixed set of scenarios as a direct result of the knapsack formulation. If for each store the R_{sj} and R'_{sj} values sufficiently converge, we can rank the objective function coefficient values and select the best K locations to receive generators for any number of generators K.



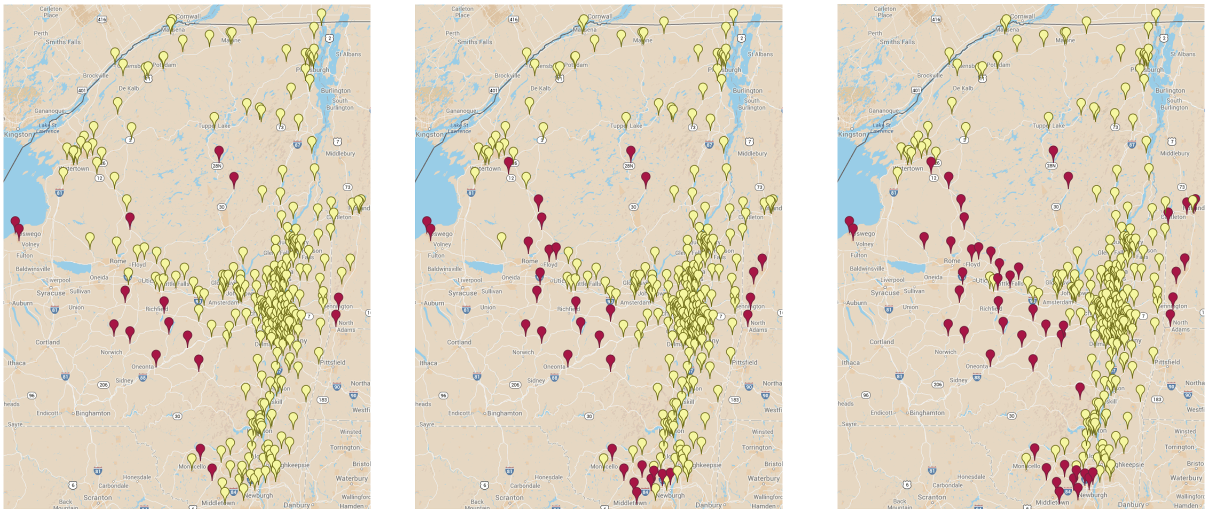
(a) 17 Generators (b) 33 Generators (c) 50 Generators

Figure 3: Hurricane Hazard Solutions



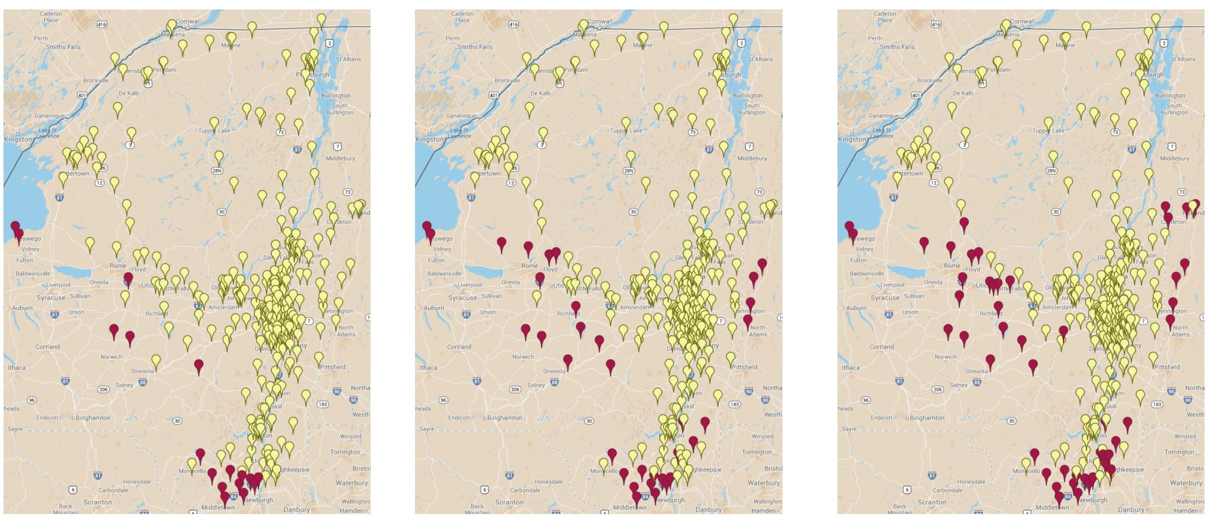
(a) 17 Generators (b) 33 Generators (c) 50 Generators

Figure 4: Flood Hazard Solutions



(a) 17 Generators (b) 33 Generators (c) 50 Generators

Figure 5: Blizzard Hazard Solutions



(a) 17 Generators (b) 33 Generators (c) 50 Generators

Figure 6: Ice Storm Hazard Solutions

Therefore, the fact that our solutions exhibit this quality, validates the use of our convergence stopping criteria. In practice, this solution property is desirable because if Stewart's Shops wanted to add more generators to their set of stores, they would not have to relocate existing generators to attain the optimal solution for the increased number. Instead they could determine which of the existing locations without generators will be selected for the installation of a generator. Also, as a direct result of the formulation, we notice the phenomenon of diminishing returns where the benefit of an extra generator decreases as the number of available generators is considered. From the figures, we see that the solutions to the hurricane, blizzard, and ice storm hazards are similar to each other. We will expand upon this observation in Section 3.3.

When examining where generators are located, we notice that many of the Stewart's Shops in rural locations were selected. As is outlined in the Appendix, the population density in the area surrounding a store impacts the time when power and communications are restored to the store, where power to urban locations is restored more quickly than to rural locations. Therefore, we see that rural locations serving a large population that have to wait longer for external services are often selected to receive generators.

3.2 Sample Average Approximation for all Hazard Types

We now seek to determine a solution that considers all four types of hazards. With these tests, we consider two different scenario sampling and weighting schematics. The first scheme looks at each type of hazard equally by incorporating an equal number of scenarios of each hazard type and distributing the weight equally across all scenarios. Specifically if $|S|$ scenarios are considered, we generate $\frac{|S|}{4}$ hurricane scenarios, $\frac{|S|}{4}$ flood scenarios, $\frac{|S|}{4}$ blizzard scenarios, and $\frac{|S|}{4}$ ice storm scenarios all with a weight equal to $\frac{1}{|S|}$. The second scheme considers each hazard equally, however, incorporates the results from Section 3.1 by generating different number of scenarios for each hazard type. Let s_i^k denote the number of scenarios needed to converge to a solution for hazard type i and generator level k (e.g. set hurricane as hazard type 1, then $s_1^{17} = 10,500$ as seen in Table 1). Under second sampling scheme, the number of scenarios generated for each type is $\frac{s_i^k|S|}{s_1^k+s_2^k+s_3^k+s_4^k}$. The weight is then assigned by hazard type, where the sum of the weights for all scenarios of a set hazard type equals 0.25. The weight for each scenario of hazard type i (assuming 4 hazard types) equals $\frac{0.25(s_1^k+s_2^k+s_3^k+s_4^k)}{s_i^k|S|}$, which is equivalent to 0.25 divided by the number of scenarios of hazard type i. We will denote the first scheme as 'Equal Sampling' and the second as 'Hazard-Based Sampling'. For both of these schemes, we continue to test for 17, 33, and 50 generators. We then use Algorithm 1 to determine when we have converged to a solution and objective function value.

Table 2 displays the computational results when all hazards are considered. We notice, that both sampling schemes need a comparable number of scenarios to converge. Further, the solutions are similar as can be seen in Figures 7 and 8 which shows the placement of the generators for the Equal Sampling and Hazard-Based Sampling tests represented by darker markers. We again see the incremental nature of the solutions as we increase the number of generators considered. For both Equal Sampling and Hazard-Based Sampling the solution with 17 generators is a subset of the solutions with 33 and 50 generators and the solution with 33 generators is a subset of the solution with 50 generators. We reinforce, that this is a nice solution property as if budgets increase to allow for the installation of more generators, the optimal solution will not require that existing generators be relocated. We explore solution similarity by examine the solutions when hazards are considered individually and collectively.

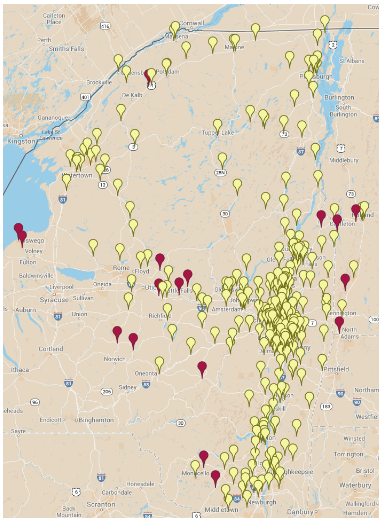
Hazard	Number of Generators (K)	Number of Scenarios for Convergence	Objective Function Value
Equal	17	12,000	9,806,580
Sampling	33	12,500	9,382,530
	50	8,000	8,994,290
Hazard-Based	17	12,000	9,820,900
Sampling	33	14,000	9,426,040
	50	9,000	8,905,650

Table 2: All Hazards Computational Results

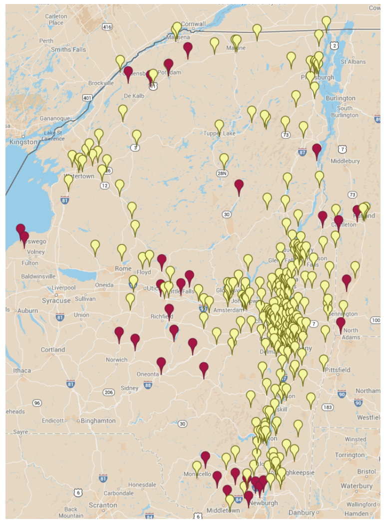
3.3 Comparison of Solutions

We now examine the solutions, i.e. where we selected to locate the generators, across the different types of hazards individually and collectively. We compare these solutions using two different metrics.

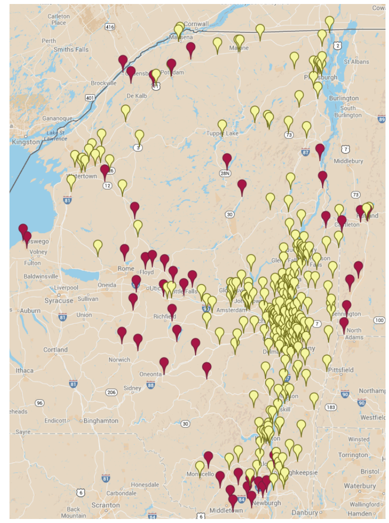
The first metric evaluates the converged solution to one test instance under s_i^k randomly selected scenarios of another test instance for hazard type i. For example, we take the



(a) 17 Generators

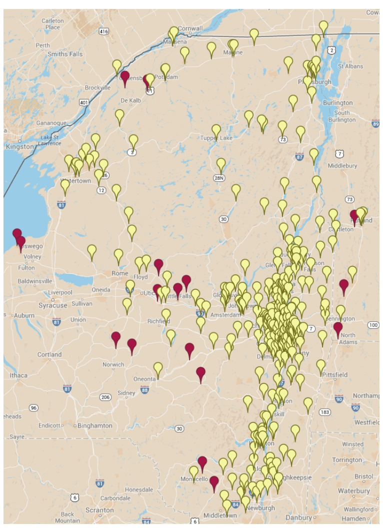


(b) 33 Generators

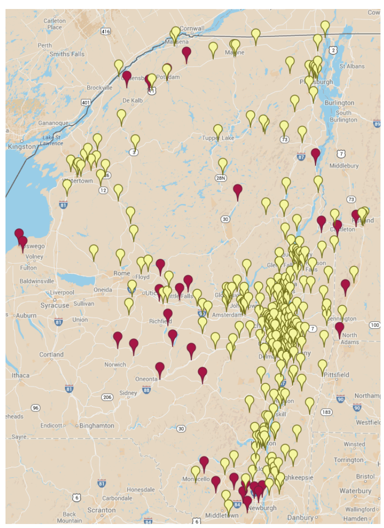


(c) 50 Generators

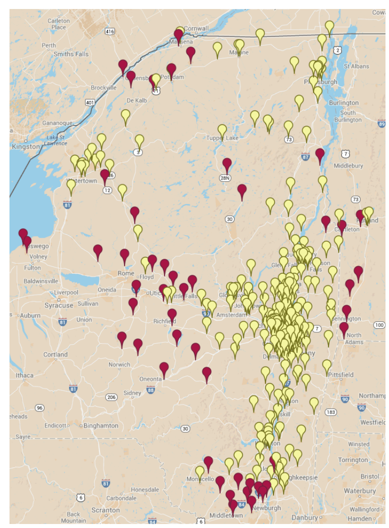
Figure 7: Solutions for All Hazards with Equal Sampling



(a) 17 Generators



(b) 33 Generators



(c) 50 Generators

Figure 8: Solutions for All Hazards with Hazard-Based Sampling

approximate optimal solution when only hurricane scenarios are considered with 17 generators and evaluate it under a test instance with 37,500 flood scenarios (from Table 1) and 17 generators. We perform this pairwise comparison for each of the hazards individually (hurricane, flood, blizzard, and ice storm) and under the two different sampling techniques (Equal Sampling and Hazard-Based Sampling). An optimality gap is then calculated as the percentage difference between the evaluated solution's objective function value and the optimal objective function value for that set of scenarios. The second metric that we use to compare solutions is a solution matching percentage which is calculated as the ratio of matching selected locations to the number of generators. For example, we take the solutions to two different problems, say only hurricane scenarios with 17 generators and only blizzard scenarios with 33 generators. We then count the number of stores that are selected to receive a generator under both solutions. This count is bounded above by the minimum number of generators considered, which in this example is 17. A solution matching percentage is then calculated by taking the ratio of the count to the minimum number of generators.

Table 3 displays the results of the comparisons using the first metric where we evaluate solutions. For each entry in the table, we create s_i^k scenarios (from Tables 1 and 2) consistent with the descriptions in the two left most columns. With this set of scenarios we perform two calculations (i) solve it to optimality (which should approximately be equal to the objective function value displayed in Table 1 or 2) and (ii) evaluate the converged solution for the type of hazard indicated by the top most row. For both calculations we capture the objective function value and calculate an optimality gap by taking the percentage difference between the evaluated solutions objective function value and the optimal objective function, which is displayed. From these results, it appears the flood hazard is least consistent with the other types of hazards as is represented by large optimality gaps. Further, it appears the Equal Sampling solution (column) performs better than the Hazard-Based Sampling solution (column) when evaluated against the different test instances as it almost always has a smaller optimality gap.

The results of the comparison using the second metric calculating a solution matching percentage are presented in Tables 4 and 5. The values are presented in Table 4 and a corresponding heat map where darker values signify closer to 1 (100%) are presented in Table 5. The calculations create a symmetric matrix, however for conciseness we leave the values below the diagonal empty. First, we note that our previous observation about the incremental nature of the solutions is verified as all values within the same hazard-hazard comparison equal 1.00 (e.g. hurricane 17 and hurricane 33). It appears that flood hazards are least similar to ice storm, blizzard, and hurricane hazards. An important point is that

Hazard	K	Hurricane	Flood	Blizzard	Ice Storm	Equal Sampling	Hazard-Based
Hurricane	17	-	4.98%	2.11%	2.69%	2.68%	4.86%
	33	-	7.15%	3.16%	3.87%	2.99%	4.13%
	50	-	9.63%	3.53%	4.41%	3.19%	4.88%
Flood	17	9.92%	-	11.31%	11.14%	4.85%	6.88%
	33	12.09%	-	13.64%	15.18%	5.91%	8.47%
	50	9.66%	-	13.37%	13.64%	6.43%	11.63%
Blizzard	17	3.85%	8.63%	-	1.92%	2.39%	5.66%
	33	5.5%	11.84%	-	1.60%	2.58%	8.35%
	50	5.56%	16.01%	-	2.07%	2.51%	7.46%
Ice Storm	17	5.88%	9.09%	2.22%	-	2.75%	5.07%
	33	9.50%	13.48%	2.22%	-	3.26%	6.29%
	50	9.78%	19.20%	2.53%	-	3.62%	6.88%
Equal	17	3.09%	3.96%	1.75%	1.60%	-	1.84%
Sampling	33	4.01%	5.78%	2.28%	1.96%	-	2.19%
	50	3.39%	9.03%	1.91%	2.14%	-	2.26%
Hazard-Based	17	5.57%	8.77%	2.64%	0.88%	2.13%	-
Sampling	33	9.74%	14.32%	2.44%	1.49%	1.88%	-
	50	9.65%	21.75%	3.17%	1.77%	2.17%	-

Table 3: Optimality gaps when the converged solution to the top row hazard is evaluated under the scenarios and optimal solution to the hazard described in the first column.

the placement of generators change significantly when we move from a single hazard to all hazards. Therefore, it is important for local supply chain distribution networks to understand their goals for their resiliency efforts and incorporate the appropriate types of hazards into their analysis. Lastly, we point out that the solutions for Equal Sampling and Hazard-Based Sampling have a very high matching percentage indicating that the biased sampling is not critical when calculating our resiliency efforts.

		Hurricane			Flood			Blizzard			Ice Storm			Equal Sampling			Hazard-Based		
Hazard	K	17	33	50	17	33	50	17	33	50	17	33	50	17	33	50	17	33	50
Hurricane	17	1.00	1.00	1.00	0.12	0.12	0.12	0.41	0.41	0.41	0.29	0.29	0.29	0.29	0.41	0.53	0.29	0.41	0.53
	33		1.00	1.00	0.24	0.18	0.18	0.59	0.33	0.36	0.29	0.24	0.24	0.35	0.39	0.48	0.41	0.39	0.48
	50			1.00	0.59	0.42	0.28	0.76	0.58	0.50	0.53	0.52	0.40	0.65	0.73	0.62	0.71	0.73	0.62
Flood	17				1.00	1.00	1.00	0.06	0.12	0.41	0.00	0.12	0.35	0.59	0.76	0.76	0.53	0.76	0.82
	33					1.00	1.00	0.06	0.09	0.27	0.00	0.09	0.24	0.59	0.42	0.55	0.53	0.42	0.58
	50						1.00	0.06	0.09	0.18	0.00	0.09	0.18	0.59	0.42	0.36	0.53	0.42	0.38
Blizzard	17							1.00	1.00	1.00	0.41	0.76	0.88	0.47	0.76	0.94	0.53	0.76	0.94
	33								1.00	1.00	0.88	0.79	0.85	0.53	0.58	0.94	0.59	0.58	0.94
	50									1.00	1.00	0.88	0.76	0.82	0.79	0.80	0.82	0.79	0.80
Ice Storm	17										1.00	1.00	1.00	0.41	0.82	1.00	0.41	0.82	1.00
	33											1.00	1.00	0.53	0.61	0.88	0.59	0.61	0.91
	50												1.00	0.76	0.73	0.70	0.76	0.73	0.70
Equal	17													1.00	1.00	1.00	0.88	1.00	1.00
Sampling	33														1.00	1.00	1.00	0.97	1.00
	50															1.00	1.00	1.00	0.96
Hazard-Based	17																1.00	1.00	1.00
Sampling	33																	1.00	1.00
	50																		1.00

Table 4: Percentage of matching solutions using the count metric.

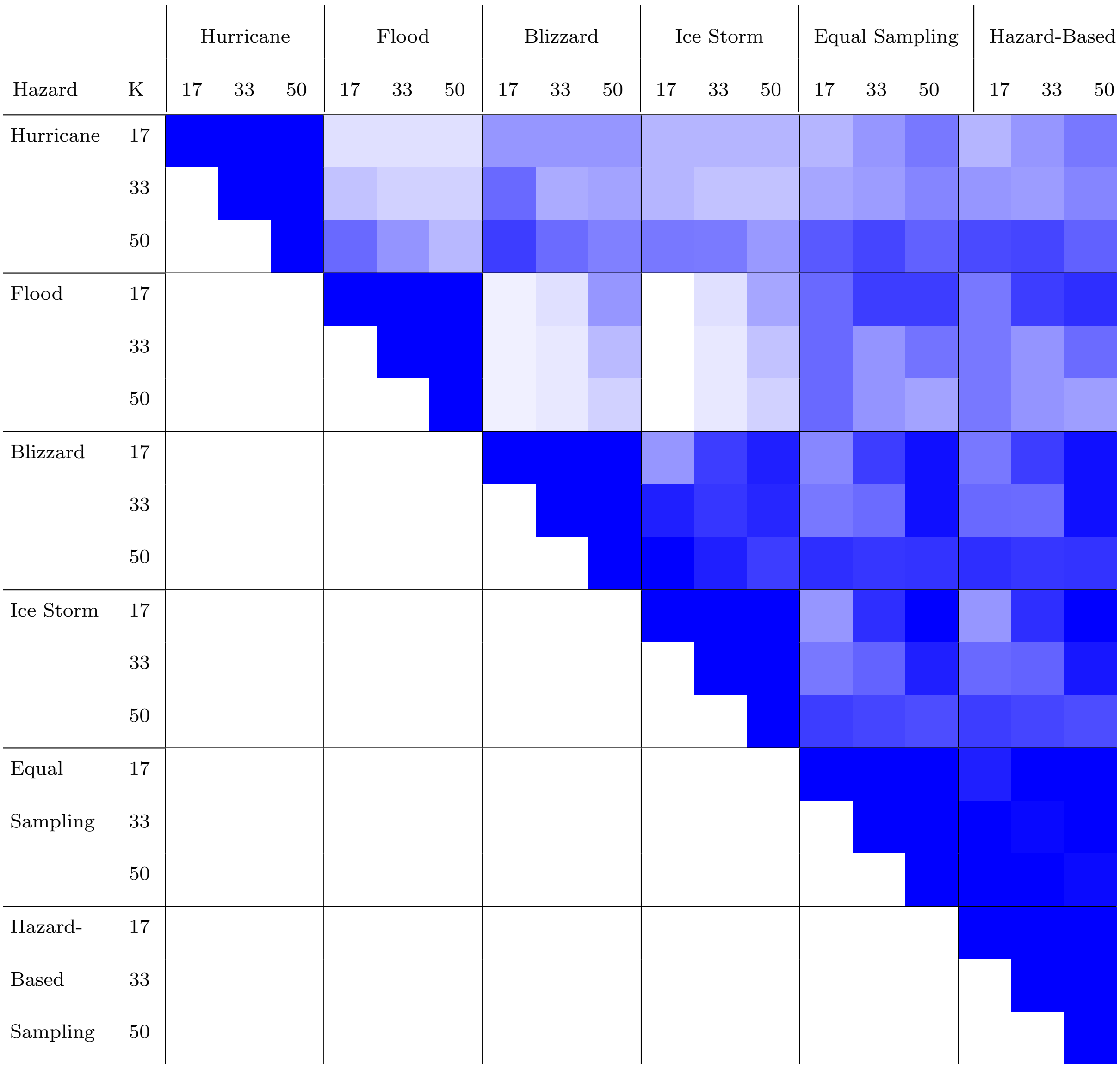


Table 5: Heat map corresponding to the values shown in Table 4 where darker represents closer to 1.

3.4 Impact of Internal and External Factors

In the last set of tests, we examine how internal and external factors impact where we locate generators. The opening time of a store depends on work being completed at the store (including tasks that require and do not require power) and the restoration of external services, such as power, communications, and transportation (e.g. flooding subsiding). We first examine the degree of impact the external factors have on the opening time of each store. To quantify this degree of impact, we perform the following procedures for each hazard and generator level. Define the opening time for a store under a 'no work' situation as the time when power and transportation is restored (for cash customers) and power, transportation, and communications is restored (for credit-only customers). (1) We first solve for the optimal 'no work' solution by (a) generating s_i^k random scenarios (from Tables 1 and 2) of hazard type i where all work is removed and (b) solving this sets of scenarios to optimality. (2) We seek to evaluate the optimal 'no work' solution under scenarios with work by (a) generating another set of s_i^k scenarios of hazard type i where all work is maintained as created, (b) solving this set of scenarios to optimality, and (c) evaluating the optimal 'no work' solution from (1) under this set of scenarios. (3) We then define the degree of impact of external factors as the optimality gap (calculated as the percentage difference) between the optimal objective function value (from (2b)) and the objective function value for the evaluated 'no work' solution (from (2c)). The placement of generators is impacted by both external factors and internal factors. External factors are present in calculations (1) and (2), while internal factors (work) is present only in calculation (2). Therefore, if external factors have a greater degree of impact the two solutions ('no work' solution and optimal solution from 2b) will be similar as is represented by a smaller optimality gap (closer to 0%). If the external factors have less impact and instead the internal factors influence the solution more greatly, the optimality gap from calculation (3) will be greater.

Secondly, we examine the impact of the opening time objective on the solution as compared to an unmet demand objective. With the opening time objective, we capture how long a customer has to wait to acquire goods from a Stewart's location. An unmet demand objective strictly looks at how many customers cannot attain goods from their Stewart's

Shop immediately following a hazardous event. These are both realistic objectives that internal management would have to consider when deciding where to locate generators among their shops. We follow a similar procedure to the one described removing work to capture the impact of switching to an unmet demand objective. We first solve a test instance to optimality with s_i^k scenarios considering an unmet demand objective. A test instance with the same hazard makeup with s_i^k scenarios is then both solved to optimality and evaluated using the optimal unmet demand solution. An optimality gap, calculated as a percentage difference, is then calculated between the optimal solution objective value and evaluated solution objective value. An optimality gap closer to 0% emphasizes that the two objectives are interchangeable.

The results of the two sets of tests are presented in Table 6. The column labeled 'External Factors' captures the degree of impact each instance has on external factors, where closer to 0% is a greater dependence on external factors. We see that the flood hazard has the greatest dependence on external factors. The result makes sense, as the flooding hazard is the only hazard that depends on the release times for flooding, power, and communications. The column labeled 'Unmet Demand' captures the interchangeability of the opening time and unmet demand objectives, where closer to 0% represents a higher degree of interchangeability. On first inspection, the optimality gaps appear not close to zero, thereby signifying that the two objectives differ. However, Figure 9 displays the 330 Stewart's locations and the count of times each location was selected to have a generator across all possible test scenarios (opening time, opening time with no work, and unmet demand), where a darker color represents a higher count. When you compare Figure 9 to Figure 2 you will see that the higher demand stores correspond to those stores with a higher selection count to receive a generator. This indicates that demand is a strong driver in the selection of generator placement under any objective. However, note that many high demand stores (particularly those near the Albany, NY area in the center right of the map) are not often selected to receive a generator. These high demand stores are in areas with highly dense populations which often have power restored quickly. It is the high demand stores in rural, less densely populated areas, that are selected to receive generators more frequently, as these stores have to traditionally wait longer for restoration of power and communications.

Hazard	K	External Factors	Unmet Demand
Hurricane	17	2.10%	3.09%
	33	2.06%	4.72%
	50	2.19%	6.95%
Flood	17	0.64%	5.86%
	33	0.49%	6.24%
	50	0.23%	2.68%
Blizzard	17	1.66%	2.54%
	33	1.60%	2.69%
	50	1.73%	4.34%
Ice Storm	17	1.13%	1.80%
	33	1.37%	1.79%
	50	1.30%	5.46%
Equal Sampling	17	1.52%	2.51%
	33	1.68%	2.84%
	50	1.45%	5.18%
Hazard-Based Sampling	17	1.66%	2.50%
	33	1.33%	3.11%
	50	1.50%	4.80%

Table 6: Optimality gaps capturing the interdependence on external services and interchangeability of different objectives.

4 Conclusions

We examined the resiliency of retail locations of a supply chain network to aid in the recovery of the local community after an extreme event. A two-stage stochastic program was used to determine the location of permanent generators among Stewart's Shops 330 stores. Consistent with recent events, we considered four types of hazardous situations that could impact Stewart's Shops: hurricanes, flooding, blizzards, and ice storms. Using an optimal

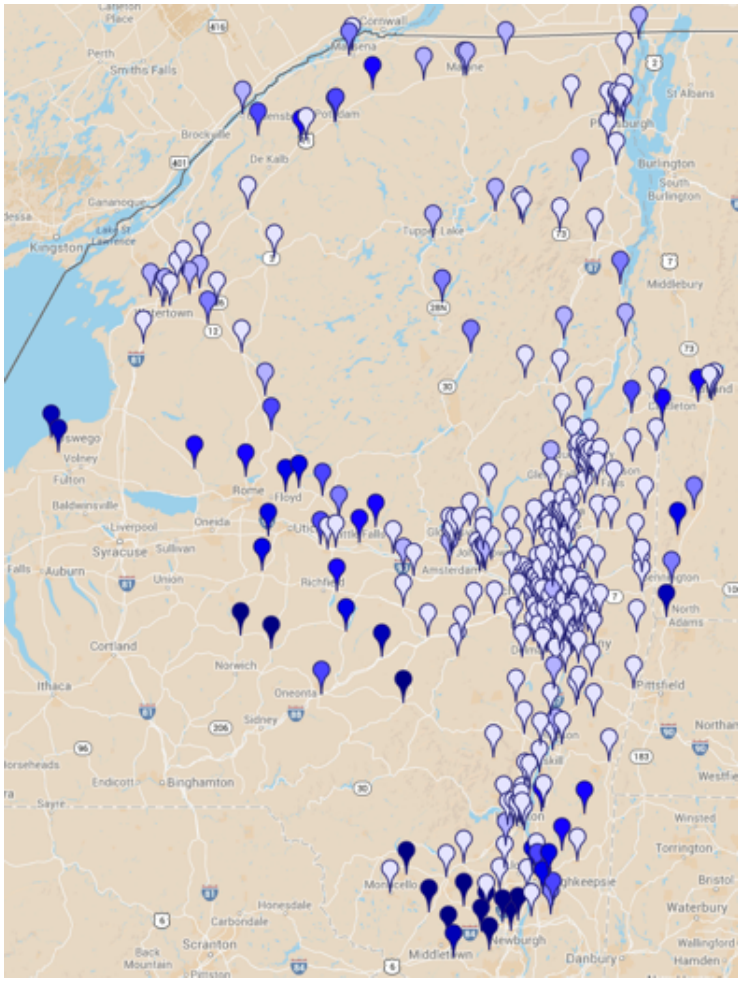


Figure 9: Heat Map displaying the 330 Stewart's Stores and their respective count of instances where they are select to receive a generator. A darker color indicates a higher count.

greedy algorithm, we tested the model for a variety of different instances by considering each hazard individually, and collectively using two different sampling procedures. We examined the impact of external factors, such as power, communications, and flooding, and different objectives on the solution obtained. The results demonstrate that we are able to empirically converge to an optimal solution, using a hybrid stopping criteria, by considering a relatively small number of scenarios. We observed that the solutions were incremental as the number of available generators increased, which is a direct result of the knapsack formulation. This is a desirable property as a prioritized list of stores that should receive generators can be made and followed as more generators become available.

Future work should involve sensitivity analysis on the parameters used to generate the hours of work that each store is required to perform both with and without power. These parameters impact the opening times of stores and ultimately if demand can be met. Further, scenario-dependent demand should be incorporated into the model. Currently, we only consider static demand but for many areas, there are many Stewart's Shops close to one another which can service customers if their preferred shop is closed.

Disclaimer

The views expressed in this article are those of the authors and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the United States Government.

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Appendix

In this section, we describe the procedures to generate a scenario for each type of hazard. For each of the four hazards (hurricane, flood, blizzard, and ice storm), we describe the different parameters that factor into the scenario generation. There are various *continuous* parameters that impact the scenario, meaning that it is not possible to generate the probability of a particular scenario based on these parameters. For each type of hazard, the parameters to consider for a scenario are:

- Hazard Characteristics: Track and intensity level (hurricane, blizzard, ice storm) or body of water (flood). Figure 10 provides the starting and ending points for hurricanes, blizzards, and ice storms. Hurricanes travel south to north and blizzards and ice storms travel west to east.

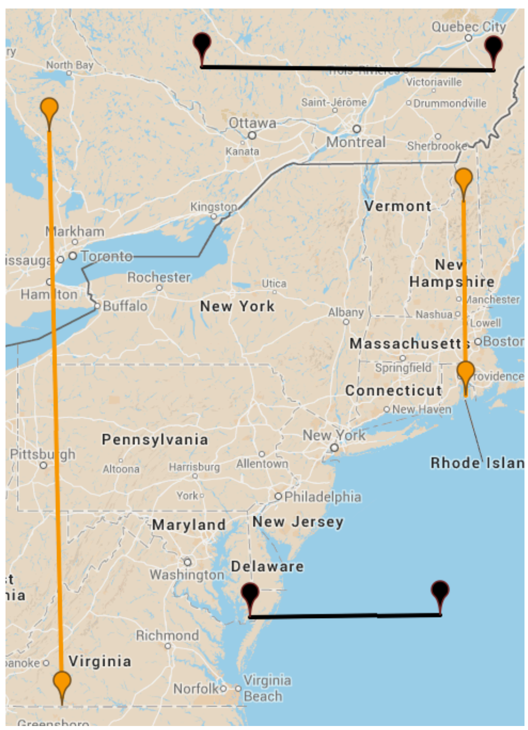

Figure 10: Potential starting and ending points for hurricane, blizzard, and ice storm tracks

- Probability of an impact for each store $j \in N$ along with whether the store is impacted for that scenario.

- Release times for power, communications, and flooding (i.e., r_{sj}^p, r_{sj}^c, and r_{sj}^f) for each impacted store $j \in N$.

- Amount of work associated with non-power tasks (i.e., $w_{sj\ell}$) for each impacted store $j \in N$ and commodity $\ell \in L$.

- Speed which non-power tasks are processed both with (σ_j^p) and without (σ_j^{np}) power available.

- Amount of time needed to process power-based tasks ($p_{sj\ell}$).

We now present the values and logic behind each of these parameters for the different types of hazards.

Hurricane

- Hazard Characteristics: We randomly sample a starting (on the south black line in Figure 10) and ending point (on the north black line in Figure 10) for the track and assume the track of the storm is a straight line between these points. We then calculate the distance from this track to each store, which we represent as δ_{sj}. Each storm has an associated intensity, $\alpha \in \{0, 1, 2\}$ where 0 represents a tropical storm, 1 represents a category 1 hurricane, and 2 represents a category 2 hurricane.

- Probability of impact for each store: The probability of a store being impacted by the storm track used in scenario s is calculated as follows:

$$p_{sj} = \begin{cases} 0.95(0.2 + \frac{\alpha}{4}) & \text{if } 0 \leq \delta_{sj} < 5 \text{ miles} \\ 0.90(0.2 + \frac{\alpha}{4}) & \text{if } 5 \leq \delta_{sj} < 10 \text{ miles} \\ 0.85(0.2 + \frac{\alpha}{4}) & \text{if } 10 \leq \delta_{sj} < 15 \text{ miles} \\ 0.80(0.2 + \frac{\alpha}{4}) & \text{if } 15 \leq \delta_{sj} < 20 \text{ miles} \\ 0.75(0.2 + \frac{\alpha}{4}) & \text{if } 20 \leq \delta_{sj} < 25 \text{ miles} \\ 0.70(0.2 + \frac{\alpha}{4}) & \text{if } 25 \leq \delta_{sj} < 30 \text{ miles} \\ 0.65(0.2 + \frac{\alpha}{4}) & \text{if } 30 \leq \delta_{sj} < 35 \text{ miles} \\ 0.60(0.2 + \frac{\alpha}{4}) & \text{if } 35 \leq \delta_{sj} < 40 \text{ miles} \\ 0.55(0.2 + \frac{\alpha}{4}) & \text{if } 40 \leq \delta_{sj} < 45 \text{ miles} \\ 0.50(0.2 + \frac{\alpha}{4}) & \text{if } 45 \leq \delta_{sj} < 50 \text{ miles} \\ 0.01(0.2 + \frac{\alpha}{4}) & \text{if } 50 \leq \delta_{sj} \end{cases} \tag{12}$$

A random number, $v_{sj} \in [0, 1]$, is generated for each store j and scenario s. If $v_{sj} \leq p_{sj}$, then we classify store j as impacted by a power outage, otherwise not. If $v_{sj} \leq \frac{p_{sj}}{3}$, we classify store j as impacted by a communications outage, otherwise not. This means that we assume that every store impacted by a communications outage is also impacted by a power outage. This is consistent with the observations after Hurricane Sandy: communications outages are typically either caused by a power outage to a local central office (implying power needs to be restored to the area) or because downed poles in the area carried both power lines and communications lines.

- Release times for power, communications, and flooding: For hurricanes, we will only consider release times for power and communications because our focus for this hazard is on wind damage (flooding from a hurricane is considered indirectly in the 'flooding hazard'). Let ξ_j denote the population density of the county of store j. The release time for power is then calculated using the following equation:

$$r^p_{sj} = 2 + \tanh\left(\frac{\delta_{sj}}{4 \cdot \xi_j}\right) \cdot 70 \tag{13}$$

which puts all power release dates in the range of $[2, 72]$ hours if the store is impacted and 0 otherwise.

The release time of communications is then calculated as a function of the release time of power and whether there is a precedence constraint between repairing power and communications. This can happen in situations when the power company owns the poles that carry telecommunications lines. If there is a power precedence, then the release time of communications is greater than the release time of power. We specifically calculate the release of communications as:

$$r^c_{sj} = \begin{cases} 0.75 r^p_{sj} & \text{if no power precedence over communications} \\ 1.25 r^p_{sj} & \text{if power precedence over communications.} \end{cases}$$

where 50% of the stores are selected at random to have a power precedence.

- Work associated with non-power tasks: For each impacted store j, we set the work for the convenience commodity to 6 hours and the work for the fuel commodity to 3 hours.

- Processing speed of non-power tasks: Work can be completed at a rate of 1 unit if there is no power and 1.5 units if there is power available (restored early or a generator).

- Time needed for power-based tasks: At each store j, we set the processing time for the fuel commodity to 2 hours. For the convenience commodity, we set the processing time to 3 hours. If a store is prone to flooding from a specific storm we add on a random integer in $[0, 15]$ to the processing time for convenience. We denote a storm prone to flooding if the store is within 10 miles of the track and within 0.1 miles of a body of water for tropical storms, 0.2 miles of a body of water for category 1 hurricanes, and 0.3 miles of a body of water for category 2 hurricanes.

Flooding

- Hazard Characteristics: There are 6 large bodies of water that could flood and impact various locations of Stewart's Shops. They are (i) Hudson, (ii) Mohawk, (iii) Lake

Champlain, (iv) Black River, (v) Vermont Rivers, and (vi) St. Lawrence river. Smaller local rivers and creeks flowing off of these bodies of water are also included in our analysis. A flood is selected uniformly at random from these 6. An intensity level, α, from $\{0.1, 0.2, 0.3, 0.4, 0.5\}$ is selected uniformly at random for each flood which represents how far away from the river flooding occurs.

- Probability of impact for each store: There is no probability associated with the impact to a store. The impact is solely determined by the flooding event and the distance from the store to the body of water. For each store j we know its distance to each of the 6 bodies of water, denote this δ_{sj} for the flooding event in scenario s. If store j is closer to the body of water than the flood intensity level ($\delta_{sj} \leq \alpha_s$) , then it is considered impacted by the event.

- Release times for power, communications, and flooding: The release time (in hours) for flooding is calculated using the table below, where the left most column represents the intensity of the flood (α_s), and the top most row represents the distance a store is from the body of water associated with s (δ_{sj}).

r^f_{sj}	0.1	0.2	0.3	0.4	0.5
0.1	20				
0.2	36	16			
0.3	50	30	14		
0.4	62	42	26	12	
0.5	72	52	36	22	10

The release time of power at store j under scenario s is calculated using the following equation:

$$r^p_{sj} = \max\left\{5 + r^f_{sj}, 5 + \tanh\left(\frac{200 \cdot \alpha}{\xi_j}\right) \cdot 150 \cdot (\alpha + 0.1)\right\} \tag{14}$$

which puts the release dates in the range $[15, 95]$ if a store is impacted and 0 otherwise.

The release time of communications is calculated using the same equation as was done for the hurricane hazard, specifically:

$$r^c_{sj} = \begin{cases} 0.75 r^p_{sj} & \text{if no power precedence over communications} \\ 1.25 r^p_{sj} & \text{if power precedence over communications.} \end{cases}$$

- Work associated with non-power tasks: For each impacted store j, we set the work for the convenience commodity to 12 hours and the work for the fuel commodity to 5 hours.

- Processing speed of non-power tasks: Work can be completed at a rate of 1 unit if there is no power and 1.5 units if there is power available (restored early or a generator).

- Time needed for power-based tasks: The processing time for power-based tasks for the fuel commodity is set to 2 hours. The processing time for power-based tasks for the convenience commodity is calculated using the table below, where the left most column represents the intensity of the flood (α_s), and the top most row represents the distance a store is from the body of water associated with s (δ_{sj}).

	0.1	0.2	0.3	0.4	0.5
0.1	10				
0.2	20	10			
0.3	30	20	10		
0.4	40	30	20	10	
0.5	50	40	30	20	10

Blizzard

- Hazard Characteristics: We randomly sample a starting (on the west orange yellow line in Figure 10) and ending point (on the east orange yellow line in Figure 10) and assume the track of the storm is a straight line between these points. We then calculate the distance from this track to each store (denoted as δ_{sj}). Each storm also has an associated intensity, $\alpha \in \{1, 2, 3, 4, 5\}$ based on Northeast Snowfall Impact Scale.

- Probability of impact for each store: Let ℓ_j denote the elevation of store j. The potential impact of the blizzard on the store is a function of its distance from the track, the intensity of the storm, and its elevation. The probability of a store being impacted by the storm track used in scenario s is calculated as follows:

$$p_{sj} = \begin{cases} 0.95(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 0 \leq \delta_{sj} < 5 \text{ miles} \\ 0.90(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 5 \leq \delta_{sj} < 10 \text{ miles} \\ 0.85(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 10 \leq \delta_{sj} < 15 \text{ miles} \\ 0.80(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 15 \leq \delta_{sj} < 20 \text{ miles} \\ 0.75(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 20 \leq \delta_{sj} < 25 \text{ miles} \\ 0.70(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 25 \leq \delta_{sj} < 30 \text{ miles} \\ 0.65(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 30 \leq \delta_{sj} < 35 \text{ miles} \\ 0.60(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 35 \leq \delta_{sj} < 40 \text{ miles} \\ 0.55(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 40 \leq \delta_{sj} < 45 \text{ miles} \\ 0.50(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 45 \leq \delta_{sj} < 50 \text{ miles} \\ 0.01(0.2 + \frac{\alpha}{10} + \min\{0.3, \ell_j\}) & \text{if } 50 \leq \delta_{sj} \end{cases} \tag{15}$$

A random number, $v_{sj} \in [0, 1]$, is generated for each store j and scenario s. If $v_{sj} \leq p_{sj}$, then we classify store j as impacted by a power outage, otherwise not. If $v_{sj} \leq \frac{p_{sj}}{3}$, we classify store j as impacted by a communications outage, otherwise not.

- Release times for power, communications, and flooding: We only consider release times for power and communications for this type of hazard. Let ξ_j represent the population density in the area surrounding store j. The release time for power is calculated using the following equation:

$$r^p_{sj} = \alpha + \tanh\left(\frac{\delta_{sj}}{4 \cdot \xi_j}\right) \cdot 50 \tag{16}$$

which puts all power release dates in the range of $[1, 55]$ hours if the store is impacted and 0 otherwise.

The release of communications is calculated using the same equation as was done for the hurricane scenario, specifically:

$$r^c_{sj} = \begin{cases} 0.75 r^p_{sj} & \text{if no power precedence over communications} \\ 1.25 r^p_{sj} & \text{if power precedence over communications.} \end{cases}$$

- Work associated with non-power tasks: For each impacted store j we set the work for the convenience commodity to 4 hours and the work for the fuel commodity to 2 hours.

- Processing speed of non-power tasks: Work can be completed at a rate of 1 unit if there is no power and 1.5 units if there is power available (restored early or a generator).

- Time needed for power-based tasks: For each impacted store j, we set the processing time for work requiring power for both the convenience commodity and the fuel commodity to 2 hours.

Ice Storm

- Hazard Characteristics: We randomly sample a starting (on the west orange yellow line in Figure 10) and ending point (on the east orange yellow line in Figure 10) and assume the track of the storm is a straight line between these points. We then calculate the distance from this track to each store (denoted as δ_{sj}. Each storm also has an associated intensity, $\alpha \in \{0, 1, 2, 3, 4, 5\}$ based on the Sperry-Piltz Ice Accumulation Index.

- Probability of impact for each store: Let ℓ_j denote the elevation of store j. The potential impact of the blizzard on the store is a function of its distance from the track, the intensity of the storm, and its elevation. The probability of a store being impacted by the storm track used in scenario s is calculated as follows:

$$p_{sj} = \begin{cases} 0.95(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 0 \leq \delta_{sj} < 5 \text{ miles} \\ 0.90(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 5 \leq \delta_{sj} < 10 \text{ miles} \\ 0.85(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 10 \leq \delta_{sj} < 15 \text{ miles} \\ 0.80(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 15 \leq \delta_{sj} < 20 \text{ miles} \\ 0.75(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 20 \leq \delta_{sj} < 25 \text{ miles} \\ 0.70(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 25 \leq \delta_{sj} < 30 \text{ miles} \\ 0.65(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 30 \leq \delta_{sj} < 35 \text{ miles} \\ 0.60(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 35 \leq \delta_{sj} < 40 \text{ miles} \\ 0.55(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 40 \leq \delta_{sj} < 45 \text{ miles} \\ 0.50(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 45 \leq \delta_{sj} < 50 \text{ miles} \\ 0.01(0.2 + \frac{\alpha}{10} + (0.3 - \min\{0.3, \ell_j\}) & \text{if } 50 \leq \delta_{sj} \end{cases} \tag{17}$$

A random number, $v_{sj} \in [0, 1]$, is generated for each store j and scenario s. If $v_{sj} \leq p_{sj}$, then we classify store j as impacted by a power outage, otherwise not. If $v_{sj} \leq \frac{p_{sj}}{3}$, we classify store j as impacted by a communications outage, otherwise not.

- Release times for power, communications, and flooding: We only consider release times for power and communications. The release time for power is calculated using the following equation:

$$r^p_{sj} = \alpha + 1 + \tanh\left(\frac{\delta_{sj}}{4 \cdot \xi_j}\right) \cdot 80 \tag{18}$$

which puts all power release dates in the range of $[1, 86]$ hours if the store is impacted and 0 otherwise.

The release of communications is calculated using the same equation as was done for the hurricane scenario, specifically:

$$r^c_{sj} = \begin{cases} 0.75 r^p_{sj} & \text{if no power precedence over communications} \\ 1.25 r^p_{sj} & \text{if power precedence over communications.} \end{cases}$$

- Work associated with non-power tasks: For each impacted store j we set the work for the convenience commodity to 2 hours and the work for the fuel commodity to 1 hour.

- Processing speed of non-power tasks: Work can be completed at a rate of 1 unit if there is no power and 1.5 units if there is power available (restored early or a generator).

- Time needed for power-based tasks: For each impacted store j, we set the processing time for work requiring power for both the convenience commodity and the fuel commodity to 2 hours.