Restoration of Services in Interdependent Infrastructure Systems: A Network Flows Approach[1]

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***Abstract* —** Modern society depends on the operations of civil infrastructure systems, such as transportation, energy, telecommunications and water. Clearly, disruption of any of these systems would present a significant detriment to daily living. However, these systems have become so interconnected, one relying on another, that disruption of one may lead to disruptions in all. The focus of this research is on developing techniques which can be used to respond to events that have the capability to impact interdependent infrastructure systems. As discussed in the paper, infrastructure interdependencies occur when, due to either geographical proximity or shared operations, an impact on one infrastructure system affects one or more other infrastructure systems. The approach is to model the salient elements of these systems and provide decision makers with a means to manipulate the set of models, i.e. a decision support system.

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Definitions of five types of interdependency identified during the research are presented and incorporated into three network flows mathematical representations. The first representation describes each system during normal operations. The second provides support to the managers of the individual systems and to emergency response officials in assessing the impact of a disruption and determining if service can be provided without extensive restoration operations. The third model shows the impact of a disruption when interdependencies among infrastructures are considered and supports strategy development and decision making during restoration. An illustrative example of the models is presented. The paper concludes with a discussion of accomplishments and opportunities for future work.

Index Terms — Civil Infrastructure Systems, Decision Support Systems, Emergency Management, Mathematical Programming, Networks.

I. BACKGROUND

Modern society relies on the operations of a set of human-built systems and their processes. The set of systems which is investigated by this research is referred to as civil infrastructure systems. These systems are typically considered to be transportation (including roads, bridges, water and rail); energy (including electric power, gas and liquid fuels); telecommunications (including telegraph, telephone, wireless and internet/digital); and finally, water (including wastewater facilities and water supplies). All civil infrastructures systems rely on a constructed system in order to provide services, such as power delivery, voice and data transmission. Each system's components can only be used to support services of their respective group (communications lines cannot be used for energy transmission and vice versa; water system pipelines are not readily available for energy products such as gas or fuel).

This set of systems is so essential that they have been called our "lifelines" [1] and is also included in the broader set of critical infrastructures defined by the President's Council on Critical Infrastructure Protection (PCCIP). As critical infrastructure systems, they are considered "so vital that their incapacity or destruction would have a debilitating effect on our defense and economic security"[2, p.3]. This research focuses on the interconnectedness of these lifeline systems. While all the systems characterized by the PCCIP report are considered critical, some of the systems, such as banking or emergency services, rely upon civil infrastructure systems in order to deliver their services. Therefore, disruption in civil infrastructure systems can cause disruption in these critical infrastructure systems, e.g., disruptions in power and communications after the 2001 World Trade Center (WTC) attack forced the closing of the New York Stock

Exchange, part of the banking and finance critical infrastructure [3]. However, this paper will focus only on interdependencies among the civil infrastructure systems. This paper first presents a discussion of infrastructures and interdependencies. The models for normal operations, response and restoration in the context of the decision environment, an activated emergency response organization, are then presented. An example of the model is provided through a case taken from accounts of the WTC attacks. The paper concludes a summary and a discussion of opportunities for future work.

A. Historical Perspective

The development of any one of these civil infrastructure systems has historically been made possible in most cases by it relying on another system [4, 5].

- Transportation is America's oldest infrastructure [6]. Horse paths and wagon trails led to rail systems stretching across the country, connecting, and in some cases creating, our cities. Within those cities, the installation of rails led to the replacement of horse-drawn omnibuses with horse-drawn trolleys [7] which were later electrified as the power systems grew. Subways appeared at the end of the nineteenth century and the twentieth century brought cars, trucks, and buses which all necessitated growth of the road infrastructure - all requiring energy systems in order to provide their service.

- Gas was the first energy infrastructure. Small, local coal gas plants produced the gas which was distributed via a dedicated piping system to homes and businesses [4, 5]. Electricity followed with its system of generators, transmission and delivery networks [4, 5]. Natural gas and petroleum pipelines and refineries complete the set of energy systems.

The pipeline systems rely on power for compressors and on communications for data acquisition and control systems.

- The growth of cities led to the need for increased water supplies. Gravity-fed or pumped water delivery from lakes, ponds and springs were followed by dams, reservoirs and the piping systems necessary to deliver the water where needed[5, 8]. Distribution networks then delivered the water to where it was needed, relying on power for pumps when gravity feed was not sufficient.

- Telecommunications was the last of the lifeline systems to appear. These systems began with the telegraph; telephones followed, evolving from operators and local switchboards to worldwide networks with high speed digital switches[5]. The internet and wireless technologies have become the newest telecommunications infrastructures.

Early power, water, sewer and gas systems were designed to serve a local populace. All such systems, with the exception of roads, were initially privately owned with customers paying for the service they received. Government at the state and federal levels took responsibility for the road systems from the beginning, using taxes and tolls to build and maintain them for the common good [5].

Each agency or company that owned or managed these systems developed its own control and monitoring systems. As the infrastructure systems grew to cover larger regions and to serve growing populations, more advanced monitoring was required. Greater efficiency was gained in systems such as communications when computers began to aid operators in decision making and control. The use of leased communication lines allowed companies to use an existing infrastructure system instead of using proprietary systems. However, reliance on another companies' systems caused interdependencies.

B. Managing Disruptions to Critical Infrastructure Systems

When an event occurs that may cause disruptions to more than one infrastructure system or is considered to be beyond the management capability of normal staff, emergency response organizations are activated. Emergency Response Organizations (EROs) exist not only at the federal, state, county or city level, but within organizations responsible for operation of the infrastructure systems [9, 10]. Immediately after the September 11, 2001 attacks in New York City, many emergency response organizations were activated. For New York City, the ERO is the Office of Emergency Management (NYCOEM); at the state level, it is the Emergency Management Office (NYSEMO); within Consolidated Edison (the principal supplier of power), it is the Corporate Emergency Response Center; for Verizon, a telecommunications provider, it is the Emergency Command Center. No matter the name, each of these emergency response organizations is established for the same basic reasons: to set priorities, coordinate response efforts, collect information and keep informed all relevant parties, both within and external to the organization [11]. For example, following the 9/11 attacks, ConEd established initial responses priorities for crews and kept NYCOEM informed. As NYCOEM became aware of needs, requests were made to responsible agencies or companies. Additionally, coordination of resources was made at NYCOEM as they were made aware of the resources each agency or company had available for response and restoration of services. When a priority was established by federal, state or city government officials, it was the responsibility of NYCOEM to make this priority clear to all member agencies.

The present research focuses on supporting the EROs in the organizations responsible for managing civil infrastructure systems in responding to events that disrupt services provided by the systems they manage. Additionally, decision support is provided to the EROs who exist at the city, county or state level in setting priorities and coordinating activities [12]. The decision makers in both types of ERO are responsible for developing strategies for response and restoration and proposing them for review by stakeholders or regulators both within and external to their organization [13]. Once a strategy has been determined, it is implemented by field personnel. The computer-based decision aid proposed in this research maintains the independent system perspective for managers of each system, while providing the interdependent view for persons charged with setting priorities and directing restoration activities when an event impacts two or more of these systems simultaneously, e.g. the New York City Office of Emergency Management.

II. INFRASTRUCTURE INTERDEPENDENCIES

In Executive Order 13010 of July 15, 1996, President Clinton established a national agenda for protecting the critical infrastructure systems. In the report of the President's Commission on Critical Infrastructure Protection, the following definition is given:

> **Infrastructure**: a network of independent, mostly privately-owned, manmade systems and processes that function collaboratively and synergistically

> to produce and distribute a continuous flow of essential goods and services [2, p. 3].

Critical infrastructures are those that are so vital that their incapacitation or destruction would have a debilitating impact on defense or economic security. The report defines Transportation; Oil and Gas Production and Storage; Water; Emergency Services; Government Services; Banking and Finance; Electrical Power; and Telecommunications as critical infrastructures [2, p. 3].

Rinaldi, Peerenboom and Kelly [14] continues the discussion of critical infrastructures and presents a conceptual framework for interdependencies that includes the following definitions:

> ***Dependency:*** A linkage or connection between two infrastructures, through which the state of one infrastructure influences or is correlated to the state of the other.
>
> ***Interdependency***: A bi-directional relationship between two infrastructures through which the state of each influences or is correlated to the state of the other. More generally, two infrastructures are interdependent when each is dependent on the other [14, p. 14].

Also, they defined four classes of interdependency:

> ***Physical*** – …a physical interdependency arises from a physical linkage between the inputs and outputs of two agents: a commodity produced or modified by one infrastructure (an output) is required by another infrastructure for it to operate (an input).
>
> ***Cyber*** – An infrastructure has a cyber interdependency if its state depends on information transmitted through the information infrastructure
>
> ***Geographic*** – Infrastructures are geographically interdependent if a local …event can create state changes in all of them.
>
> ***Logical*** – Two infrastructures are logically interdependent if the state of each depends on the state of the other via a mechanism that is neither physical, cyber nor geographic connection [14, p.14-16].

Due to the number of different types of dependencies and interdependencies, Rinaldi, Peerenboom and Kelly [14] classifies the entire family of interrelationships between systems as interdependencies, an approach retained in this paper. The objective of the definitions provided by both PCCIP and Rinaldi, Peerenboom and Kelly is to aid in the discussion of policies for addressing the vulnerability of infrastructures to natural, technological and intentional human-induced hazards[2, 14]. However in order to develop models designed to provide decision support to emergency managers, these definitions must be further refined.

In this research, an *infrastructure* is defined as a linked set of physical components with associated activities. *Physical component*s are the built part of an infrastructure; *activities* are tasks necessary to operate physical components of the infrastructure. An *intersection* is the area where two or more physical components meet or are joined. An intersection circumscribes the activities and physical components necessary to manage the connection between the joined physical components. As an example, the intersection of two roadways may have one or more physical components (e.g., a traffic signal) and activities (e.g., manipulation of the signal via sensors embedded in the roadway). All intersections in a given infrastructure must have a physical component.

A *service* is something made available by the infrastructure for use or consumption. A service may be used by people or by other infrastructures: it is provided in order to meet a real or perceived need. An infrastructure can provide one or more services. *Material* is any physical entity or “substance or substances out of which a thing is or can be made” [15, p.837]. Examples include electrons, people, product, and electromagnetic signals. Provision of a service requires activities such as movement, collection, transformation or storage of material. Activities may be initiated at one or many locations and may be terminated at one or many locations. Assuming that traversal of a connection between two intersections requires a set of activities from beginning to end, management activities are necessary when provision of the service requires traversal of more than one intersection.

A *disruption* in an infrastructure is said to occur when one or more of the physical components or one or more of the activities needed to operate a physical component cannot function at prescribed levels. Disruption may or may not result in service degradation. *Service degradation* is said to occur when the service itself cannot be provided at its prescribed level.

The current research identifies five types of interrelationship between infrastructure systems:

- *Input*: the infrastructure requires as input one or more services from another infrastructure in order to provide some other service.
- *Mutually dependent*: at least one of the activities of each infrastructure in a collection of infrastructures is dependent upon each of the other infrastructures. (An example of mutual dependence involving two infrastructures occurs when an output of infrastructure A is an input to infrastructure B, and an output of infrastructure B is an input to infrastructure A.)
- *Co-located*: any of their physical components are situated within a prescribed geographical region.
- *Shared (AND)*: some physical components or activities of the infrastructure used in providing the services are shared.
- *Exclusive-or* (*XOR*): only one of two or more services can be provided by an infrastructure. (Note that a disturbance in an infrastructure that is dependent on another by virtue of its inability to operate if the other infrastructure is operating will effect just its own provision of service.)

Collectively, these five conditions—input, mutual dependence, co-location, shared and exclusive-or—will be denoted types of *interdependence*, since all imply that an impact on one infrastructure system is also an impact on one or more other infrastructure systems. These definitions of civil infrastructure systems and their interdependencies form the basis of the mathematical models developed in the next section.

III. The Modeling Paradigm: Network Flows

Interdependent infrastructures are viewed as networks, with movement of commodities (i.e. material) corresponding to flows and with services corresponding to a desired level of these flows. For ease of representation, each network, or infrastructure system, is defined as a collection of nodes and arcs with commodities flowing from node to node along paths in the network. Activities, physical components and intersections are considered to be contained within a node. Similarly, management activities are not considered in traversal of an arc; they are contained within the arc itself. For each commodity, each node is either a supply node which is a source for the commodity; a demand node which is a point that requires some amount of the commodity; or a transshipment node which is a point that neither produces nor requires the commodity but serve as a point though which the commodity passes [16]. Arcs may, of course, have limited capacities [17]. Infrastructure systems operate in an environment subject to disruptions, natural, human-caused or willful acts. Based upon performance criteria, an infrastructure system can be designed to minimize possible service degradation following a disruption. In addition, once a disruption occurs, alternative ways of restoring service can be determined.

A. Normal Operations Model

Mathematically, a collection of infrastructure systems is represented as follows. Let I denote the set of infrastructures. Infrastructure $i \in I$ has nodes V^i and directed arcs E^i. Associated with each node $j \in V^i$ is a scalar b^i_j representing its supply or demand. If node $j \in V^i$ is a demand point then $b^i_j < 0$; if it is a supply point then $b^i_j > 0$; and if it is a transshipment node then $b^i_j = 0$. If

$j \in V^i$ is a supply node then b^i_j equals the maximum possible amount that could be produced at that node. A nonnegative vector of variables, x^i_e, represents the flow on each arc e of the infrastructure. Associated with each arc e in E^i are non-negative scalars of costs c^i_e and capacities u^i_e, where $0 \le x^i_e \le u^i_e$.

Arcs are represented using either the endpoints of the arc or the index of the arc. For a node $l \in V^i$ for some infrastructure $i \in I$, let $\delta^+(l)$ denote the set of arcs in E^i that enter node l and let $\delta^-(l)$ denote the set of arcs in E^i that leave node l. Define $\delta(l) := \delta^+(l) \cup \delta^-(l)$, the set of all arcs incident to node l. Without loss of generality, assume that every supply node has no incoming arcs (i.e., $\delta^+(l) = 0$ if $b^i_l > 0$) and that demand nodes have no outgoing arcs, (i.e., $\delta^-(l) = 0$ if $b^i_l < 0$). A transshipment node j may have a limited capacity, w^i_j, modeled by placing an upper bound on total flow across the arcs $\delta^+(l)$. Included in the model are *flow conservation constraints* (i) that for supply nodes ensure that total flow out of the node is no greater than the available supply, (ii) that for demand nodes ensure that demand is met, and (iii) that for transshipment nodes ensure that flow into the node equals flow out of the node. The structural requirements are modeled by constraints on the capacities of arcs and transshipment nodes.

The objective during normal operations of a civil infrastructure system is to find the minimum cost feasible network flow. The complete representation of minimum cost network flow for each infrastructure $i \in I$, where the total flow into node j is given by $\sum_{e \in \delta^+(j)} x^i_e$ and the total flow out of the node is given by $\sum_{e \in \delta^-(j)} x^i_e$, is as follows:

$$\text{minimize} \qquad \sum_{e \in E^i} c_e^i x_e^i \tag{1}$$

$$\text{subject to} \qquad \sum_{e \in \delta^-(j)} x_e^i \le b_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i > 0 \tag{2}$$

$$\sum_{e \in \delta^+(j)} x_e^i = -b_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i < 0 \tag{3}$$

$$\sum_{e \in \delta^+(j)} x_e^i - \sum_{e \in \delta^-(j)} x_e^i = 0 \qquad \text{for } j \in V^i \text{ with } b_j^i = 0 \tag{4}$$

$$\sum_{e \in \delta^+(j)} x_e^i \le w_j^i \qquad \text{for } j \in V^i \text{ with } b_j^i = 0 \tag{5}$$

$$x_e^i \le u_e^i \qquad \text{for } e \in E^i \tag{6}$$

$$x_e^i \ge 0 \qquad \text{for } e \in E^i \tag{7}$$

Under normal conditions, all demands of all infrastructures are met. Referring back to the definition of input dependency, if all demands are met then all the interdependent components operate. So when all demands are met, the systems can be looked at as operating independently. It is only when failures occur that interdependencies become a concern. This *normal operations model* provides the baseline representation of an infrastructure.

B. Response to a Disruption

When an incident occurs that has the potential to cause a major disruption in an infrastructure, initial activities include assessing the (i) impact on physical components of infrastructure systems, (ii) potential loss of service, (iii) impact on the safety of humans and (iv) effect on the security of sensitive systems in the natural and built environments. Identification of suspect components could be aided by the human-machine interface of a decision support system, for example, a Geographic Information System (GIS) [18]. An operator is able to identify those components in or near the event area. Based on reports from field observers and experience of system managers, alterations in capabilities of infrastructure components are made. These assessments include operating conditions and capacities. Assessment of new demands is also made, since post-event conditions can result not only in decreases but in increases in demand for services. The impact assessment results in a reconfigured network with revised flows for each infrastructure system directly affected by the disruption.

The impact assessment may reveal that demand levels cannot be met. However, once demand levels are ascertained and prioritized, it may be possible to satisfy the revised demands using functioning supply points that, prior to the event, had been operating at less than full capacity. Absent such a situation, the model for normal operations must be revised by changing supplies, demands and capacities of components. Infrastructure operators and emergency managers must also be able to identify instances of unmet demand within a particular infrastructure system.

To be of use in addressing disruptions, the normal operations model is reformulated by the addition of slack variables and the capability to weight these variables so that emergency managers in the Emergency Response Organization (ERO) may prioritize. These weighting factors cause the model to attempt to reduce a priority demand's slack to zero first, before meeting demands with lower priority. Therefore the *response model* is given by: for infrastructure $i \in I$, where the total flow into node j is given by $\sum_{e \in \delta^+(j)} x_e^i$ and the total flow out of the node is given by $\sum_{e \in \delta^-(j)} x_e^i$,with s_j^i , as slack variables and weighting factors k_j^i, the *response model* is as follows:

$$\text{minimize} \quad \sum_{e \in E^i} c_e^i x_e^i + \sum_{j \in V^i} k_j^i s_j^i \tag{8}$$

$$\text{subject to} \quad \sum_{e \in \delta^-(j)} x_e^i \le b_j^i \quad \text{for } j \in V^i \text{ with } b_j^i > 0 \tag{9}$$

$$s_j^i + \sum_{e \in \delta^+(j)} x_e^i = -b_j^i \quad \text{for } j \in V^i \text{ with } b_j^i < 0 \tag{10}$$

$$\sum_{e \in \delta^+(j)} x_e^i - \sum_{e \in \delta^-(j)} x_e^i = 0 \quad \text{for } j \in V^i \text{ with } b_j^i = 0 \tag{11}$$

$$\sum_{e \in \delta^+(j)} x_e^i \le w_j^i \quad \text{for } j \in V^i \text{ with } b_j^i = 0 \tag{12}$$

$$x_e^i \le u_e^i \quad \forall i \in I \text{ and } \forall e \in E^i \tag{13}$$

$$x_e^i \ge 0 \quad \forall i \in I \text{ and } \forall e \in E^i \tag{14}$$

$$s_j^i \ge 0 \quad \forall i \in I \text{ and } \forall j \in V^i \tag{15}$$

where the symbolic representations are the same as the those for the normal operations model.

The response model identifies the unmet demands by component or location. However, due to system interdependencies, an operator of one infrastructure system may not realize the impact of this unmet demand on the other systems. Therefore, decision support for the managers in the ERO responsible for coordinating among infrastructure systems, e.g. New York City Office of Emergency Management (NYCOEM), must be able to identify and display unmet demands that cause interdependencies to become critical, (i.e. prevent delivery of service due to disruption of one or more infrastructure systems). Once the need for developing ways of restoring service has been determined, managers of the various infrastructure systems are contacted and physical and personnel resources available for implementing restoration strategies identified.

C. Interdependencies and the Restoration Model

As previously noted, if all revised demands for all infrastructure services can be met, each infrastructure system is considered to be operating independently. However, when unmet demand for any infrastructure service is found, interdependencies among infrastructure systems are considered and incorporated in order to support the restoration decision-making process.

The material in the remainder of this section describes how the various interdependencies defined in Section II are represented in the restoration model.

1) Input: An infrastructure is input dependent when it requires as input one or more services from another infrastructure in order to provide some other service. As an example, in the case of a telephone switching station, the switching station itself is a transshipment node within the telecommunications network. However, this same switching station from the perspective of the electrical network is seen as a demand node since it needs an adequate source of electricity to operate. From the perspective of the electrical network, the switching station is therefore a dependent component. This situation may be represented more formally as follows. Denote the demand node for the switching station in the electrical network to be node j. If there is an adequate flow of electric power into node j, the switching station can function; otherwise, the switching station fails. A binary variable, y, is used in this case to represent the two states of the switching station. If adequate power is available at j, then $y = 1$; if not, then $y = 0$. The phone switching station also has some maximum capacity within the telecommunications network. The station's capacity can be represented as the product of the binary variable y and the rated capacity. When adequate power is available the station can operate to its rated capacity (since $y = 1$). On the other hand, if adequate power is not available then the capacity of the station is 0. This binary variable y serves as a virtual connector between the two systems. Its value is set by the conditions existing in one system, and affects the operating characteristics of a second system. Events affecting the power network that have an effect on node j in turn impact a node in the model of the telecommunications network. The effect on any set of systems can be analyzed in a similar manner. Note that some interdependent infrastructure system failures may result in reducing capacity to some value other than zero. For example, loss of supervisory control systems in a subway system may result in operators exercising greater care and slowing trains. So the post-disruption capacity may be lower than normal. In this case, the connector variable y would shift from 1 to a lower value. The exact effect of each disruption must be evaluated during impact assessment.

In general, input dependency is represented as follows: Define the set $V^{i,+} \subseteq V^i$ to be the nodes $j \in V^i$ with $b_j^i > 0$ (supply nodes). Sets $V^{i,=} \subseteq V^i$ (transshipment nodes) and $V^{i,-} \subseteq V^i$ (demand nodes) are defined similarly. Let $D(i, i_1) \subseteq V^{i,-}$ be the set of nodes in i that some other infrastructure i_1 depend upon (parent nodes) and let $D^i := \cup_{i_1 \in I, i_1 \neq i} D(i, i_1)$. This subset of nodes is the interdependent nodes. The remaining nodes in $V^{i,-}$ will be referred to as the independent nodes. The binary variable $y_{i_1,j}^{i,l}$ is the connection between node l in infrastructure i (where it is a demand node) and node j in infrastructure i_1, where it may be either a supply, demand or transshipment node and is only defined for $l \in D(i, i_1)$.

Let $C(i_1, i) \subseteq V^{i_1}$ be the set of nodes in i_1 that depend on some other infrastructure i, (child nodes) and let $C^{i_1} := \cup_{i \in I, i \neq i_1} C(i_1, i)$.Without loss of generality, all nodes have been disaggregated to the point where, given infrastructures i, i_1, and l in $D(i,i_1)$, there is a unique node

j in $C(i_1,i)$ such that $y_{i1,j}^{i,l}$ is defined, and given infrastructures i, i_1, and node j in $C(i_1,i)$, there is a unique node l in $D(i,i_1)$, such that $y_{i1,j}^{i,l}$ is defined. Let $F(i,i_1)$ be the set of ordered pairs (l,j) associated with node l in $D(i,i_1)$ and node j in $C(i_1,i)$ for each $y_{i1,j}^{i,l}$.

The objective function of the restoration model incorporates different priorities in addition to modeling interdependencies. On independent nodes, the available supply may be meeting the required demand or there may be some shortfall. The slack variable s_j^i represents the shortfall in meeting demands at independent nodes. In the model, there is no consideration for partial slack at the interdependent nodes. Because these interdependent nodes control the operation of nodes in other infrastructure systems, if they are not fully operational then they are in a failed condition: there is no benefit to partially meeting the requirement. Following the response phase, when there are unmet demands across one or more systems, one choice for the objective function is to minimize the total shortfall (slack) plus the unmet interdependent demands. A *restoration model* is defined as follows:

$$\text{minimize} \qquad \sum_{i \in I} \sum_{j \in V^{i,-} \setminus D^i} k_j^i s_j^i + \sum_{i \in I} \sum_{l \in D^i} \sum_{i_1 \neq i} b_l^i (1 - y_{i_1,j}^{i,l}) \qquad (16)$$

subject to

$$\textstyle\sum_{e \in \delta^-(j)} x_e^i \le b_j^i \qquad \forall j \in V^{i,+}, \forall i \in I \qquad (17)$$

$$\sum_{e \in \delta^+(j)} x_e^i - \sum_{e \in \delta^-(j)} x_e^i = b_j^i \qquad \forall j \in V^{i,=}, \forall i \in I \qquad (18)$$

$$s_j^i + \sum_{e \in \delta^+(j)} x_e^i = -b_j^i \qquad \forall j \in V^{i,-}, \forall i \in I \qquad (19)$$

$$\sum_{e\in\delta^+(j)} x_e^i \le w_j^i \qquad \forall j \in V^{i,=}, \forall i \in I \tag{20}$$

$$\sum_{e\in\delta^-(j)} x_e^{i_1} \le b_j^{i_1} y_{i_1,j}^{i,l} \qquad \forall (l,j) \in F(i,i_1) \text{ with } b_j^{i_1} > 0, \forall i, i_1 \in I, i \ne i_1 \tag{21}$$

$$s_j^{i_1} + \sum_{e\in\delta^+(j)} x_e^{i_1} = -b_j^{i_1} y_{i_1,j}^{i,l} \qquad \forall (l,j) \in F(i,i_1) \text{ with } b_j^{i_1} < 0, \forall i, i_1 \in I, i \ne i_1 \tag{22}$$

$$\sum_{e\in\delta^+(j)} x_e^{i_1} \le w_j^{i_1} y_{i_1,j}^{i,l} \qquad \forall (l,j) \in F(i,i_1) \text{ with } b_j^{i_1} = 0, \forall i, i_1 \in I, i \ne i_1 \tag{23}$$

$$s_l^i \le (1 - y_{i_1,j}^{i,l}) b_l^i \qquad \forall (l,j) \in F(i,i_1), \forall i, i_1 \in I, i \ne i_1 \tag{24}$$

$$x_e^i \le u_e^i \qquad \forall e \in E^i, \forall i \in I \tag{25}$$

$$x^i \ge 0 \qquad \forall i \in I \tag{26}$$

$$y_{i_1,j}^{i,l} \qquad \text{binary}, \forall (l,j) \in F(i_1,i), \forall i, i_1 \in I, i \ne i_1 \tag{27}$$

$$s_j^i \ge 0 \qquad \forall j \in V^i \text{ with } b_j^i < 0, \forall i \in I \tag{28}$$

For the remaining four interdependencies, their mathematical representations are as follows.

2) Mutual Dependence: A collection of infrastructures is said to be mutually dependent if at least one of the activities of one infrastructure system is dependent upon any other infrastructure system and at least one of the activities of this other infrastructure system is dependent upon the first infrastructure system. So in the case of two systems *i* and i_1, mutual dependence would occur if there is at least one $y_{i_1,j}^{i,l}$ (connection between node l in infrastructure i (where it is a demand node) and node j in infrastructure i_1) and at least one $y_{i,n}^{i_1,m}$ (connection between node *m* in infrastructure i_1 (where it is a demand node) and node *n* in infrastructure *i*). Consider a natural gas system pump and a gas-fired electric power generator. From the perspective of the natural gas system, the pump is a transshipment node and the generator is a demand node. From the perspective of the electrical network, the generator is a supply node and the pump is a demand node. The generator needs gas to produce electricity; the pump needs electric power to deliver gas through the system to the generator. In this case $y_{i_1,j}^{i,l}$ would be the connection from the power system (infrastructure *i*) to the pump in the gas system (infrastructure i_1), and $y_{i,n}^{i_1,m}$, the connection from the gas system (infrastructure i_1) to the generator in the power system (infrastructure *i*). Failure of one component causes its corresponding binary variable to be set to zero, thus reducing the effective capacity of the other component to zero. In other words, if the pump were to fail, supply of gas to the generator would be inadequate and $y_{i,n}^{i_1,m}$ would be set to zero. When $y_{i,n}^{i_1,m} = 0$, the capacity of the generator is set to zero (since its effective capacity is the product of $y_{i,n}^{i_1,m}$ and its rated capacity b). Because the generator is a supply node, all flows on the arcs (i.e., the power lines) leaving the generator would now be zero, by flow conservation. Alternately, a lack of power at the pump demand node in the electrical generating network causes its binary variable $y_{i_1,j}^{i,l}$ to be set to zero and the capacity of the pump to be set to zero. To correct this situation, either an alternate source of gas must be found for the generator or an alternate source of power must be found for the pump.

3) Co-located: The co-located interdependency occurs when any of the physical components or activities of the civil infrastructure systems are situated within a prescribed geographical region. It was previously noted that managers of individual infrastructure systems would identify the components of their respective system at or near the site of the incident which may have been affected by the event. Based on further investigation, the status of these components will be adjusted. However, since only those EROs who are responsible for coordinating activities across multiple agencies maintain the complete view of all civil infrastructure systems, it is ultimately their responsibility to ensure that all co-located interdependencies have been considered and the models of the affected infrastructures revised as appropriate.

4) Shared (AND): Shared interdependence occurs when some physical components and/or activities of the infrastructure used in providing the services are shared. As an example, after the World Trade Center attack ferries were used as ambulances for medical transport and for commuters [19]. The use of the one or more shared components for more than one commodity or activity is constrained by a limit on maximum flow. In the context of the telecommunications and power systems and the WTC restoration, it could have been advantageous to route the shunts used to restore phone and power through the same temporary enclosures. This might have reduced time and cost since only one enclosure would need to be built, but could then lead to coordination problems between the two companies. This situation could be modeled by changes in the objective function and constraint equations and will be shown in the illustrative example.

5) Exclusive-or (XOR): When multiple services share infrastructure component(s), for example, an arc, but the component can only be used by one service at a time, exclusive-or interdependence occurs. In the first few days following the WTC attacks, streets (i.e., shared components) could not be used by both the emergency response personnel and financial district workers. This conflict had to be resolved prior to reopening the New York Stock Exchange [20]. Considering power and telecommunications, it can also be the case that a power and a telecommunications shunt may not be able to be routed in close proximity to each other. This would be the case with a T-1 line and a high voltage distribution line which can not be too close together due to RF interference considerations. Exclusive-or interdependencies are modeled by selecting additional constraints to restrict flow to one commodity or the other. The representation of the exclusive-or will also be shown in Section IV.

An illustrative example of the use of the model in the context of post disruption response and restoration is presented in the following section. This example is based upon ongoing research concerning the events of September 11, 2001.

IV. An Illustrative Example

A. Background

This section presents an example drawn from cases of infrastructure interdependency that arose following the World Trade Center attack, as reported by *The New York Times*. Additional information was obtained from interviews with Consolidated Edison (ConEd) and Verizon personnel. Much of the data associated with the attack (e.g., locations of equipment and personnel, generating capabilities, capacities of feeder lines and shunts, power demands) is sensitive and has not been used. In order to illustrate modeling of each of the five interdependencies, a simulated event is used, a monitoring system (telecommunications) failure leading to losses of power.

> In Lower Manhattan, about 300 Con Edison workers are trying to restore service to about 12,000 commercial and residential buildings without electricity,

1,400 without gas and nearly 300 without steam, said Michael Clendenin, a spokesman for the utility. Mr. Clendenin said there was no estimate on when power would be restored….

Electrical, gas and steam service were normal throughout the city, except for the areas affected by the loss of two substations that were knocked out when 7 World Trade Center caught fire and collapsed. Water and sewer service were not affected outside the area from Broadway to the Hudson River between Murray and Rector Streets. But many people outside the downtown area were affected by Verizon's problems.

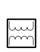
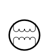
At least 200,000 of the roughly 500,000 phone lines south of 14th Street remained out of service, although most of those lines served locations that are either not in use or no longer exist, said Peter Thonis, another Verizon spokesman. Overall, there are about 2 million phone lines in Manhattan. The 500,000 lines in southern Manhattan are served by five Verizon switching centers, one of which is on West Street near the location of 7 World Trade.

That switching center lost power, affecting not only the 200,000 phone lines but also about three million private data lines for corporate customers. Those lines include about 20 percent of the data lines that serve the New York Stock Exchange.

> Upper levels of the West Street office, filled with communications equipment, were damaged by the twin towers' fall and were penetrated by steel girders when 7 World Trade Center collapsed. But beyond direct physical damage, perhaps Verizon's biggest problem on West Street is broken water mains.
>
> Even more than the West Street office, however, the New York Stock Exchange depends on a Verizon switching center on Broad Street that handles about 80 percent of the exchange's data lines. That office was not physically damaged by the blasts but lost power from Con Ed shortly afterward. Diesel generators there have continued to provide power, and power was restored to the site late Wednesday... [21, p A-12]

Motivated by the foregoing, the following case illustrates how the response and restoration models could be used to provide decision support to infrastructure operators and emergency managers.

B. Scenario

The first system used in this case is the power distribution system. This system as depicted in Figure 1 shows four high voltage power supplies, shown as . This high voltage is the input to substations, shown as , which transform down the high voltage power received from the transmission system to 13,500 volts (13.5 kV). From these substations, power is provided to 120/208 volt transformers, shown as  and then to the customers. The customers depicted include the New York Stock Exchange (NYSE), a hospital, One Police Plaza (the New York

City Police Headquarters), two facilities of the Metropolitan Transportation Authority (transit services), six Controlled Environmental Vaults (CEV) of the phone company (described below), and two general residential areas and two general business areas.

Insert Figure 1 here (attached at end)

The second system is the telephone system, in which customers function as both supply and demand nodes. The system depicted in figure 2 shows 18 neighborhood areas consisting of both residential and business customers. Calls originate at a customer and are collected along a distribution cable typically serving dozens of customers. Many distribution cables come together at a Controlled Environmental Vault (CEV). Calls then pass through a feeder cable containing thousands of lines and come together in the cable vault of a central office and into a switching system. From the central office, they pass to one of the following: to another central office through an interface trunk; to a tandem[1] via a trunk link; or out through the same set of CEVs that feed the (originating) central office. In figure 2, the CEVs are connected to Central Office A. Central office A is connected to Central Office B (in an adjacent area) and the Tandem.

Insert Figure 2 here (attached at end)

To illustrate a mutual dependence of telecommunications on power and power on telecommunications, assume a failure in the power distribution system which causes the failure of a Controlled Environment Vault (CEV) in the phone system. The failure of the CEV results in

[1] A tandem has trunk lines to all central offices in its sector and trunks to all other tandems with the same of other companies providing service to the world network.

a loss of telephone service. With this failure in the phone system, the supervisory control and data acquisition (SCADA) system for the power company becomes unreliable, causing loss of reliable indicators on a set of distribution transformers and causing breakers to malfunction. (As noted earlier, this failure was not observed during the WTC attack and is only inserted to illustrate mutual dependency.) The disruption causes the failure of substation 1, the power supply line to CEV D and CEV E , and the phone feeder lines from CEVs B, C, and D to Central Office A.

C. Impact Assessment

As part of the impact assessment, field observers report that substation 1 is completely destroyed. They also report that they are unable to ascertain the condition of the four lines that were affected due to the extensive debris but are confident that the lines are not serviceable. The system operators at Verizon and ConEd make modifications to the response phase model, using data from the field. The supply at substation 1 and the capacity of the one electric and three phone lines are reduced to zero. Each manager runs the response model on his or her system independent of the other.

The results show unmet demand at the New York Stock Exchange, residential areas, CEV C, a hospital, CEV D and CEV E based on the slack variables in the power system model. In the phone system, the operator notes there is unmet demand in the neighborhoods served by CEVs B, C, and D. In the response model discussed earlier, it is noted that prioritization can be done in an attempt to meet vital loads at the expense of less important loads. However, in the power and phone systems contained in this model, no alternate paths exist in the system. Therefore, prioritization can have no effect on restoring loads to service.

With unmet demands in their respective systems and no feasible solutions, the operators provide these data to the emergency response organizations that then move the event to the next stage – restoration. The operator uses the restoration model to identify the slack variables corresponding to unmet demands in the two systems, including interdependencies. Since the effects on these two systems are being considered, the input dependencies are the loss of power to CEV C and CEV D. CEV E contains the SCADA system for substation 2. Following the loss of power to CEV B, the loss of SCADA causes a loss of reliable indication and control of the output breakers causing them to open resulting in loss of power to all components served by substation 2 — a mutual dependence.

D. Restoration Phase

When the emergency response organization runs the restoration model, the results indicate the unmet demands noted by the infrastructure managers as well as several new, unmet demands. The loss of power to CEV C via its input dependence results in loss of service in the neighborhoods it serves. Failure of CEV E and the SCADA system leads to new failures in the power grid and unmet demand at Metropolitan Transportation Authority (MTA) trains and stations (subway services) and One Police Plaza (police headquarters). NYCOEM personnel now move to the second portion of the restoration phase as depicted in figures 3 and 4). Available resources are identified and restoration strategies are developed, in consultation with the individual system infrastructure managers. These strategies consist of new lines and temporary power sources.

Insert figures 3 and 4 here (attached at end)

Priority for the power company is restoring power to the New York Stock Exchange, CEV C, CEV D, CEV E, and the hospital (restoring power to CEV E will restore SCADA to substation 2 and therefore, restore power to One Police Plaza and the MTA facility). The power company has also received a request to provide new power lines to the area of the disruption for rescue operations (lighting, pumps, etc). The power company collects the following information for the model. Substation 2 is available to provide 125 units of power beyond its current loads (once SCADA is restored and power is provided to One Police Plaza and MTA); substation 3 has 50 units of power available for restoration efforts beyond its current loads and substation 4 can provide 60 units beyond its current loads. CEV C, CEV D and CEV E each require 10 units of power; the New York Stock Exchange requires 100; the hospital requires 125 and the World Trade Center site needs 50.

The phone company is focused on restoring the lines between CEVs B, C, and D and the Central Office. All of the temporary lines run must be housed in enclosures to ensure the safety of the public and to protect the lines from damage. Phone shunts which contain only voice circuits, known as POTS, can be housed in the same enclosure as the power line. However, any phone line containing a T-1 line must not be run in the vicinity of a power line, due to interference. These requirements serve as the bases for the remaining two interdependencies, shared (AND) and exclusive-or (XOR). The POTS lines and power are modeled as a shared interdependency; that is, the lines will be run together when possible. The T-1 line and power line are an exclusive-or interdependency, where only one line or the other may be routed along a particular path.

The power company has sufficient shunts to connect from substations 2, 3 or 4 to each of the loads. Each shunt from a supply to a demand node has its associated cost and capacity. There are also two diesel generators available and four suitable sites for them. Each site could have one or both generators, allowing for two one-generator sites or one two-generator site.

The phone company has sufficient resources to reconnect the three CEVs to the central office. Due to the location of the failure, the company also has an option to connect to another central office versus the original connection. There are multiple routing choices available for each connection, each having its own associated cost. However, because this portion of the restoration is being done in conjunction with the power company, both the XOR constraint of only one system's line being located along some paths and the AND constraint of both systems' lines being in the same enclosure must each be taken into account.

Based on discussions with domain experts, reasonable cost estimates are presented in Table 1. Each cell represents the cost of the shunt to connect from a power source to a demand site.

TABLE I

POWER SHUNT COSTS

Power Sources	Demand Sites					
	NYSE	Hospital	CEV C	CEV D	CEV E	WTC Site Power
Substation 2	165	155	185	110	80	80
Substation 3	105	95	85	115	130	90
Substation 4	190	180	220	140	115	115
Generator site 1	65	55	45	75	90	60
Generator site 2	10	90	130	80	95	55
Generator site 3	65	55	95	25	10	40
Generator site 4	165	155	195	115	90	90

Similarly, the phone company has the following possible shunts, with their respective costs and interdependencies.

TABLE II

PHONE SHUNT COSTS AND INTERDEPENDENCIES

From	To	Cost	Interdependencies
CEV B	Original CO	65	XOR interdependency with Electric shunt from diesel site 2 to CEV D
CEV B	Alternate CO	75	
CEV C	Original CO	175	
CEV C	Alternate CO	145	
CEV D	Original CO	105	AND interdependency with Electric Shunts from Substation 4 to either CEV D or E (cost savings of 40 to COA)
CEV D	Alternate CO	135	AND interdependency with Electric Shunts from Substation 4 to either CEV D or E (cost savings of 60 to COB)

The decision situation facing the emergency managers is, in essence, to construct a new network utilizing the working sections of the infrastructures and supplementing them with new shunts and temporary diesel generators. The specific objective function for this example is to minimize cost of operation of the shunts and the generators.

E. Formulation

The restoration model is designed to support selection of a restoration strategy. The constraints have been specified in terms of available resources and unmet demand; the objective function needs to be specified. Each shunt has fixed cost, q_e^i, and power cost, c_e^i, which is a function of generator or transformer use at the substations. The cost of power is set at 1 for power coming from the distribution grid and to 3 for power from generators. The cost to operate each shunt is therefore $c_e^i x_e^i + q_e^i$, where x_e^i is the amount of power flowing on the shunt. Using binary variables r_e^i to indicate whether or not a shunt is installed, $r_e^i = 1$ when $x_e^i > 0$. If it is desirable to use power from a diesel site *j*, then the diesel installation incurs an addition fixed cost, d_j. In this example, d_j is constant at 25 for each generator installation. Similarly, a binary variable t_j indicates if the diesel is installed, and $t_j = 1$ when any r_e^i from a diesel site equals 1 (r_e^i=1 implies that the shunt is installed and in use). Phone shunts also have cost q_e^i and binary variable r_e^i to indicate when one is installed. To take advantage of the discounts p_k in meeting an AND interdependency, a binary variable z_k indicates when the AND constraint is met (The set of shunts having AND interdependencies will be referred to as *k*) . There is also a binary variable *g* which limits the number of diesel sites to two if one-diesel sites are built and one if a two-diesel site is built (*g* is 1 when a two diesel site is built). The objective is to minimize the total cost of installing and operating all shunts, as follows:

minimize $$\sum_{e\in E^i}(c_e^i x_e^i + q_e^i r_e^i) + \sum_{j\in V^i} d_j t_j - \sum_k z_k p_k \qquad (28)$$

subject to $\sum_{e\in\delta^-(j)} x_e^i \le b_j^i$ for $j \in V^i$ with $b_j^i > 0$ (29)

$\sum_{e\in\delta^+(j)} x_e^i - \sum_{e\in\delta^-(j)} x_e^i = b_j^i$ for $j \in V^i$ with $b_j^i \le 0$ (30)

$r_e^i \le t_j$ for $e \in \delta^-(j)$ for those *j* with diesels (31)

$x_e^i \le 350 r_e^i$ $\forall e \in E^i \quad i = power$ (32)

limiting the number of one diesel sites to two at most:

$\sum_j t_j \le 2 - (2 * g)$ for *j* with one diesel (33)

limiting the number of two diesel sites to one at most:

$\sum_j t_j \le 1 * g$ for *j* with two diesels (34)

Installing only one phone shunt between each CEV and Central Office

$\sum_{e\in\delta^-(j)} r_e^i = 1$ for *i*= phone; *j* = each CEV (35)

where the set $\delta^-(j)$ is the set of all proposed phone shunts from the CEVs to the Central Offices.

The XOR constraints for each respective pair:

r (for respective electric shunt) + *r* (corresponding phone shunt) ≤ 1 (36)

The AND constraint for each respective pair (each AND pair will be subscripted *j*)

r (for respective electric shunt) $\ge z_j$ (37)

r (corresponding phone shunt) $\ge z_j$ (38)

$x_e^i \ge 0$ $\forall e \in E^i, i \in I$ (39)

$$r_e^i, t_j, g, z_j \qquad \text{binary.} \qquad (40)$$

F. Course of Action

Utilizing the modeling language AMPL and the CPLEX solver [22], the model determines the following course of action: From substation 2, connect a shunt to the New York Stock Exchange (NYSE). From substation 3, connect one shunt to CEV C and a shunt to the hospital to provide part of its loads. From substation 4, connect a shunt to the WTC site and connect another to CEV E. Place two diesels at site 3 and supply the remaining needs of the hospital and CEV D. The phone company runs its shunts from CEV B to the original central office and from CEV C and D to the alternate office. This plan meets the needed demands and does not violate the exclusive or requirements.

The model can also provide decision makers with alternatives. The effects of changes in the current situation can be evaluated. This is usually referred to as sensitivity analysis [16]. For example, if the loads increase, the current solution could become infeasible (it will no longer meet the requirements). Reducing loads could lead to new, cheaper solutions. How much of a change is required to affect the current solution is determined by the sensitivity analysis. Utilizing the CPLEX sensitivity option, the current solution is infeasible if: 1) either CEV C or CEV D require more than 15 units; 2) there is any increase in the demand of CEV E; 3) the hospital increases above 130 units; 3) the New York Stock Exchange increases beyond 105 or; 4) the World Trade Center site requires more than 50. New, cheaper solutions are possible if: 1) CEV C or D reduce to 0; 2) the hospital decreases to 40; 3) the New York Stock Exchange decreases to 0 or; 4) the World Trade Center site requires less than 50. Managers could also use sensitivity analysis and new constraints to handle other contingencies such as more generators becoming available or shunts above some length (cost) should not be installed.

V. Summary and Significance

A. Summary

Models can provide powerful means of understanding [23], monitoring and controlling large-scale infrastructure systems [24]. The need for powerful but parsimonious models is particularly acute as modeled infrastructures increase in complexity, as when a number of infrastructures are interdependent. The particular focus of this work is on developing techniques that can be used to respond to and restore from events that have the capability of impacting interdependent infrastructure systems. The approach taken is to model salient elements of interdependent critical infrastructure systems and to provide decision makers with means of manipulating this model for purposes of response and restoration of service, i.e. a decision support system.

Definitions of various types of infrastructure interdependencies were developed and incorporated into a mathematical representation of interdependent infrastructure systems. This representation permits the development and use of algorithms for identifying solutions to problems associated with disruptions to interdependent critical infrastructures. The models allow representation of infrastructures under conditions of normal operations, post-disruption impact assessment and finally, restoration.

B. Opportunities for Future Work

1) Decision Support System: These models are designed to be imbedded in a decision support system that will employ a database management system for storing data and information on response and restoration resources and have as the human-machine interface, a geographical information system. Emergency managers and infrastructure operators will then be able to see the full impact of actions across multiple systems and work collaboratively to provide the solutions that are best for all. Additionally, it is envisioned that this decision support system will have the capability of aiding system designers in increasing the resilience of their systems and increasing their awareness of the effect interdependency plays in the design and operation of these complex systems.

2) Time-Expanded Networks: Some effects of a disruption in service take time to develop. For example, a generator may be able to produce additional power to cover a shortfall for just a limited amount of time and additional resources may be available in the future. These slow-moving consequences of the disruption should appear in the model. Therefore, the model should consider the state of the system at different points in time, since we have a time-varying network flow. The graph can be expanded by taking snapshots corresponding to different points in time. The set V^i is replaced by multiple copies $V^{i,t}$, and similarly for other sets. Arcs can be added to link the different snapshots in order to represent temporal relationships. This results in a time-expanded network [17]. The sets can evolve over time, as either extra nodes become unavailable or as nodes are repaired. The available arcs can change over time as emergency power lines are laid, for example.

3) Algorithmic Choices: Solution procedures will be able to take advantage of the structure of the network formulation. Our models have a number of network constraints, which can be exploited to speed up solution both of the linear programming relaxations (through the use of specialized linear programming algorithms, based on either the simplex method [17] or on interior point methods [25] and of the integer programming problem (through exploitation of the total unimodularity property of network constraint matrices). Algorithms available for solving the integer programming problem include branch-and-cut [26] for recent surveys), branch-and-price [27], and the use of Lagrangian relaxation. These approaches use constraint and/or column generation; for larger problems, interior point methods for solving the linear programming relaxations become considered [28, 29].

C. Conclusion

The anticipated results of the research will improve society's ability to withstand the impact of and respond to events that can disrupt the provision of services that are required for the health, safety and economic well being of the citizenry. Managers of critical infrastructures and emergency response officials will be able to model different event scenarios and assess their impact on the services provided by critical infrastructure systems. With this knowledge, mitigation and preparedness strategies can be formulated and evaluated for their ability to prevent an emergency from escalating into a disaster and, if a disaster does occur, ensure a rapid restoration of critical services.

APPENDIX
GLOSSARY OF SYMBOLS

$i,\ i_1$	Infrastructure systems in the set *I*
I	The collection of all infrastructure systems
b_j^i	The supply or demand at node *j* in infrastructure *i*
V^i	The complete set of nodes in infrastructure *i*
$V^{i,+}$	The set of supply nodes in *i.*
$V^{i,=}$	The set of transshipment nodes in *i.*
$V^{i,-}$	The set of demand nodes in *i.*
$V^{i,t}$	The complete set of nodes in infrastructure *i* at time *t*
e	An arc in infrastructure *i*
E^i	The complete set of arcs in infrastructure *i*
x_e^i	The flow on arc *e* in infrastructure *i*
c_e^i	The cost associated with flow along arc *e* in infrastructure *i*
u_e^i	The capacity of arc *e* in infrastructure *i*
$\delta^+(l)$	The set of arcs that enter node *l*
$\delta^-(l)$	The set of arcs that leave node *l*
$\delta(l)$	The set of arcs incident to node *l*
w_j^i	The capacity of node *j* in infrastructure *i*
s_j^i	The slack associated with node *j* in infrastructure *i*
k_j^i	Weighting factor for node *j* in infrastructure *i*
D^i	The set of all nodes in *i* upon which any other infrastructure nodes depend
$D(i,i_1)$	The set of nodes in *i* that some other infrastructure i_1 depend upon
C^{i_1}	The set of all nodes in i_1 which depend on any other infrastructure nodes
$C(i_1,i)$	The set of nodes in i_1 that depend on some other infrastructure *i,*
$F(i,i_1)$	The set of ordered pairs (*l,j*) associated with node *l* in $D(i,i_1)$ and node *j* in $C(i_1,i)$
$y_{i_1,j}^{i,l}$	The connection between node *l* in infrastructure *i* (where it is a demand node) and node *j* in infrastructure i_1

Specific Variables from the Case:	
q_e^i	Fixed cost associated with installing shunt *e* in infrastructure *i*
r_e^i	Variable indicating whether or not shunt *e* is installed in infrastructure *i*
d_j	Fixed cost associated with installing a diesel generator at site *j*
t_j *t*	Variable indicating whether or not a diesel is installed at site *j*
p_k	Discount associated with AND interdependency *k*
z_k	Variable indicating when AND interdependency *k* is met
g	Variable indicating whether a one- or two-diesel site has been built

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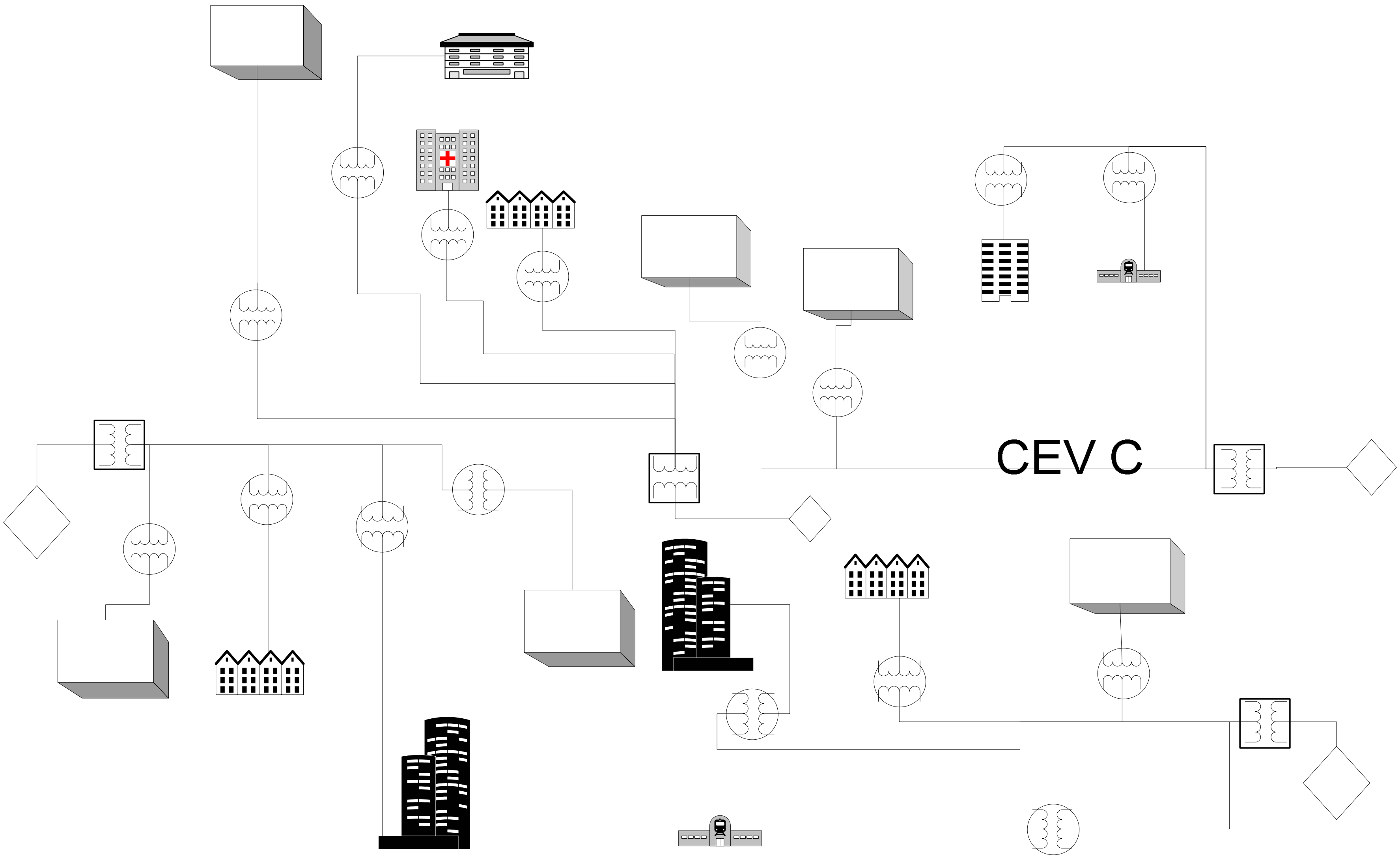

Fig. 1. Power distribution system for the case.

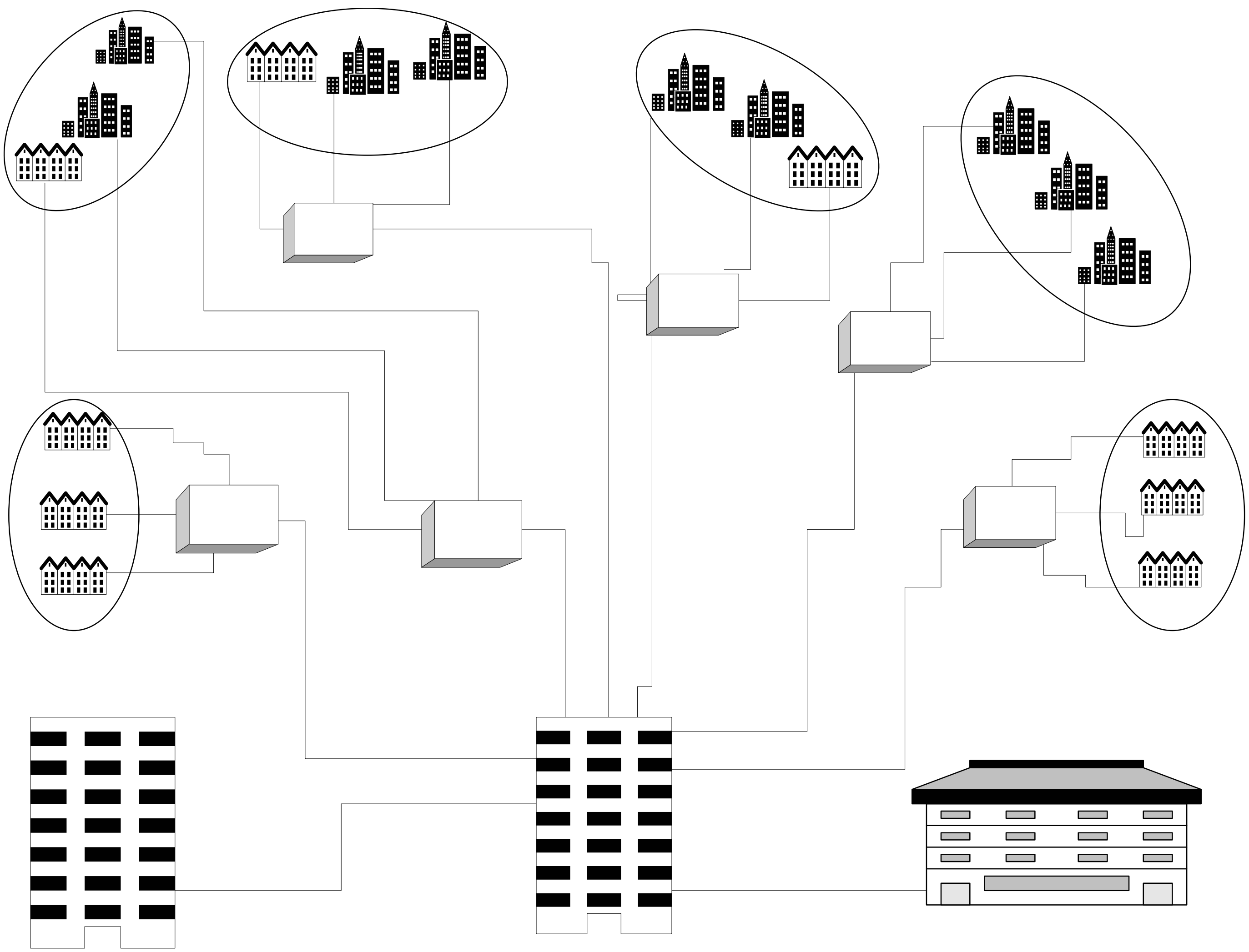

Fig. 2. Telecommunications system for the case.

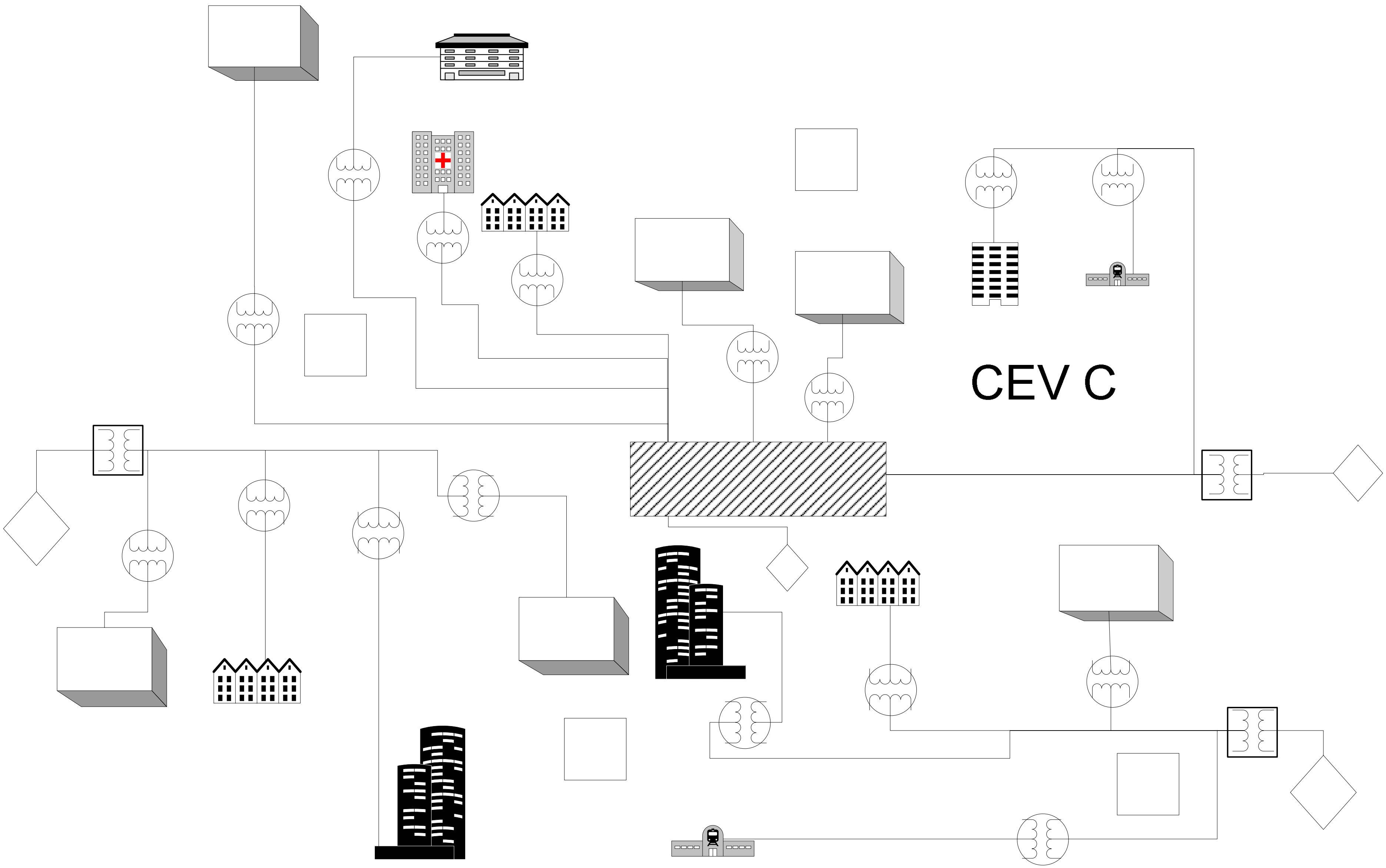

Fig. 3. Post-disruption power distribution showing affected area and proposed diesel locations.

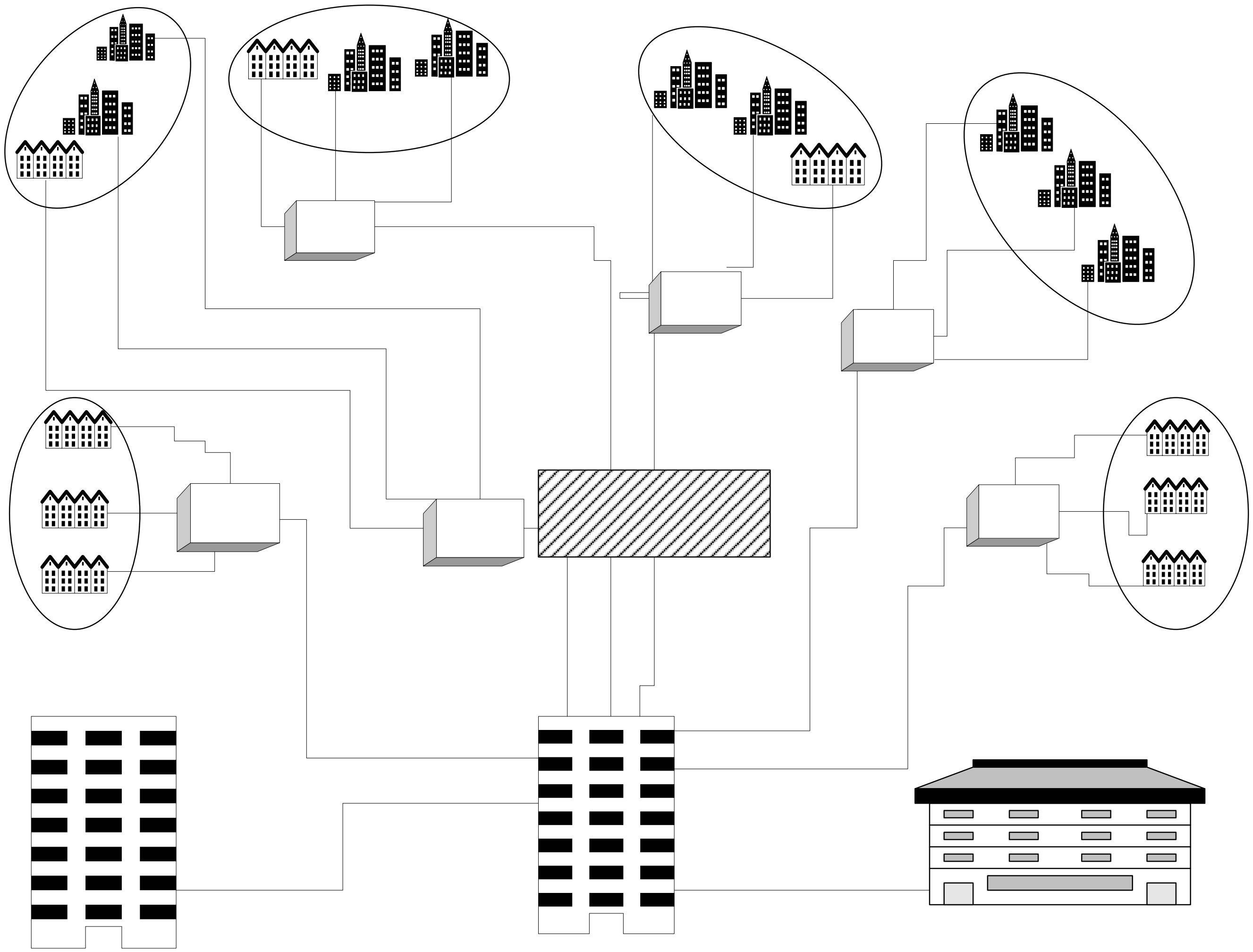

Fig. 4. Post-disruption telecommunications system showing affected area.

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